

# Aggregate Implications of a Credit Crunch\*

Francisco J. Buera<sup>†</sup>

Benjamin Moll<sup>‡</sup>

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## Abstract

We assess the common practice of trying to learn about the sources of aggregate fluctuations using aggregate data and a representative agent framework. We do this in the context of financial frictions and study the mapping from a credit crunch, modeled as a shock to collateral constraints in a heterogeneous agent economy, to simple aggregate wedges in a representative agent economy. Does a credit crunch show up as an investment wedge? Or as an efficiency wedge? Or even as a labor wedge? We find that a credit crunch worsens the allocation of resources across heterogeneous individuals and does not affect aggregate investment in the sense that the Euler equation of a representative entrepreneur is undistorted. Furthermore, we show that depending on the form of underlying heterogeneity a credit crunch manifests itself as any of an efficiency, investment or even a labor wedge. Attempts to understand the role of financial frictions in business cycles using a representative agent framework and aggregate data alone can therefore be very misleading.

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<sup>†</sup>UCLA and NBER, [fjbuera@econ.ucla.edu](mailto:fjbuera@econ.ucla.edu).

<sup>‡</sup>Princeton University, [moll@princeton.edu](mailto:moll@princeton.edu).

# Introduction

What are the sources of aggregate fluctuations? To answer this question, macroeconomists often rely on aggregate data and the representative agent framework, thereby abstracting from underlying heterogeneity in the economy. One common approach is to use aggregate productivity shocks, preference shocks, or more generally wedges on the optimality conditions of the representative agent to account for aggregate fluctuations. An obvious advantage of this approach is its simplicity. Financial frictions are one potential source of aggregate fluctuations and the representative agent framework has been used to infer their relative importance.<sup>1</sup> To assess the usefulness of this approach, we study the mapping from a credit crunch, modeled as a shock to collateral constraints in a heterogeneous agent economy, to aggregate variables in a representative agent economy. If such a mapping exists and is unique, it provides a justification for the representative agent approach we just described. To be more concrete, we examine the mapping from a credit crunch to simple aggregate wedges and ask whether a credit crunch shows up as an investment wedge, as an efficiency wedge, or even as a labor wedge.

We find that a credit crunch worsens the allocation of resources across heterogeneous individuals, but does not affect aggregate investment in the sense that the Euler equation of a representative entrepreneur is undistorted. The aggregate implications of a credit crunch depend crucially on where an economy features heterogeneity. A credit crunch shows up as an efficiency wedge if there is heterogeneity in the productivity of final goods producers. In this case, labor and investment wedges are zero, or unimportant. In contrast, a credit crunch shows up as an investment wedge if we replace heterogeneity in the productivity of final goods producers with heterogeneous investment costs. In this specification, TFP and labor wedges are unaffected. Finally, a credit crunch shows up as a labor wedge in an economy with heterogeneous recruitment costs. Hence we find that, while there exists a simple mapping from a credit crunch in a heterogeneous agent economy to the aggregate variables in a representative agent economy, this mapping depends crucially on the underlying heterogeneity. It is therefore necessary to explicitly model the heterogeneity that creates a need for financial intermediation to understand the role of financial frictions in business cycle fluctuations. Simple shortcuts can be very misleading.

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<sup>1</sup>For instance, Smets and Wouters (2007) use aggregate time series and various structural shocks, including a risk premium shock and an investment-specific technology shock, to understand the sources of business cycle fluctuations. Chari, Kehoe and McGrattan (2007) argue that aggregate data and wedges are a useful guide for researchers when developing quantitative models of aggregate fluctuations, and that the investment wedge accounts for only a very small fraction of these fluctuations. Other examples include Ohanian (2010), Justiniano, Primiceri and Tambalotti (2010) and Justiniano, Primiceri and Tambalotti (2011).

We develop a unified framework with heterogeneous entrepreneurs facing collateral constraints. Entrepreneurs have access to three constant returns to scale technologies: a technology to produce final goods, another technology to transform final goods into capital, and a technology for transforming recruitment effort today into workers in the following period. Entrepreneurs are potentially heterogeneous in their productivity of each of these technologies. They also face collateral constraints limiting the fraction of their capital stock or future revenues that can be pledged to collateralize loans. Collateral constraints limit entrepreneurs' ability to acquire capital and recruit workers. Entrepreneurs also have the option to invest in a risk-less bond. They will choose to invest in the risk-less bond if their productivity is low relative to the risk-free rate.

Entrepreneurs' productivities evolves stochastically. In each period they draw a new productivity from a fixed distribution. This assumption guarantees that the distribution of capital across individuals of different productivities is not degenerate, and therefore, collateral constraints bind in equilibrium. We assume that the productivity of the final good technology is known a period in advance, so that the return to investment is deterministic at the moment an entrepreneur invests.

In addition to entrepreneurs, the economy is populated by a continuum of homogeneous individuals who supply labor. We consider two alternative assumptions regarding their access to asset markets: the case of financial autarky and the case where they are allowed to save in the risk-free bond. The first assumption allows for a sharper theoretical characterization of the model's transitional dynamics.

We first study a model economy in which entrepreneurs are heterogeneous in the productivity of their final goods technology, but do not differ in their investment or recruitment technologies. We show that a credit crunch is isomorphic to a TFP shock for the case of logarithmic preferences. Even though individual investment decisions are distorted, we show that the aggregate behavior of entrepreneurs can be characterized in terms of the Euler equation of a representative entrepreneur that is not distorted by a credit crunch. This result is due to a general equilibrium effect: the interest rate adjusts in such a way that a credit crunch only affects the allocation of capital but not the aggregate demand for capital. While a credit crunch distorts individual returns to savings and hence individual Euler equations, this effect is entirely absorbed by a decrease in TFP. While this result is exact only for the case of logarithmic utility, we show by means of numerical simulations that it holds approximately for the case of general Constant Relative Risk Aversion preferences under standard parameter values.

Once we aggregate entrepreneurs the economy consist of two types of agents, a representative entrepreneur and a representative worker who is in financial autarky. Given the heterogeneity across these two groups of agents, and our assumption that workers are in financial autarky, an investment wedge is needed to characterize aggregate data in terms of a representative agent. We show that the aggregate investment wedge is small and negative: in the aggregate, a credit crunch is an episode in which investment appears to be subsidized, not taxed (and aggregate productivity is low). In addition, we show by means of simulations that the magnitude of the investment wedge is small if workers face idiosyncratic labor risk and accumulate a risk-less assets.

Having studied our first model with heterogenous final goods productivity, we consider two models with heterogeneity along two other dimensions. In the second model economy entrepreneurs face heterogenous investment costs – meaning they differ in their technologies to transform final goods into investment goods – but are homogeneous in their final goods production and recruitment technologies. In the third model economy entrepreneurs face heterogenous recruitment costs – meaning they differ in their technologies to transform recruitment effort today into workers in the following period.

In these models a credit crunch shows up as an investment wedge and a labor wedge respectively. In our three different models, we therefore obtain three different results on the wedges implied by a credit crunch. The logic behind these results is, however, exactly the same. In particular, all three results are direct implications of the result that a credit crunch worsens the allocation of resources across heterogenous individuals and does not directly affect aggregate investment. For instance, in the case of heterogenous investment technologies, a credit crunch leads to a worse aggregate investment technology, which will be interpreted as an investment wedge. A similar intuition applies to the model with heterogeneous recruitment technologies.

**Related Literature** Following Bernanke and Gertler (1989), a large theoretical literature studies the role of credit market imperfections in business cycle fluctuations. Most papers are similar to ours in that they study heterogenous entrepreneurs subject to borrowing constraints. In light of our finding that the exact form of heterogeneity matters, we note that most of them assume that entrepreneurs are heterogenous in their investment technologies. For instance, Carlstrom and Fuerst (1997), Kiyotaki and Moore (1997), Bernanke, Gertler and Gilchrist (1998), Kiyotaki and Moore (2005), Kiyotaki and Moore (2008), Gertler and Kiyotaki (2010) all make the assumption that each period “investment opportunities” arrive randomly to some exogenous fraction of entrepreneurs. Only entrepreneurs with an “investment opportunity” can

acquire new investment goods; others cannot.<sup>2</sup> Models with entrepreneurs that are heterogenous in their final goods productivity are rarer. Exceptions are the papers by Kiyotaki (1998), Kocherlakota (2009), Khan and Thomas (2010), Bassetto, Cagetti and De Nardi (2010) and Brunnermeier and Sannikov (2011).<sup>3</sup>

Our paper and the majority of the literature focus on credit constraints faced by non-financial borrowers (entrepreneurs) who, in effect, borrow directly from other non-financial lenders. Recent papers by Gertler and Karadi (2011) and Gertler and Kiyotaki (2010) give a more prominent role to financial intermediation and instead postulate that it is intermediaries themselves that face balance sheet constraints when obtaining deposits. We also focus on the implications of a tightening of collateral constraints for the production side of an economy. Exceptions that study borrowing constraints at the household level are Guerrieri and Lorenzoni (2011) and Midrigan and Philippon (2011).

Also closely related, of course, is the literature using wedges in representative agent models to summarize aggregate data. Prominent examples examining the efficiency wedge, investment wedge as well as the labor wedge in a common framework are Mulligan (2002) and Chari, Kehoe and McGrattan (2007).<sup>4</sup> Chari, Kehoe and McGrattan find that the investment wedge did not fluctuate much over the business cycle in postwar aggregate data. They show that in some of the popular theories cited above, financial frictions manifest themselves primarily as an investment wedge and conclude that those theories are not promising for the study of business cycles. In contrast, Justiniano, Primiceri and Tambalotti (2010) and Justiniano, Primiceri and Tambalotti (2011) view the data through the lens of a “New Neoclassical Synthesis” model instead of an RBC model, and argue that most business cycle fluctuations are driven by shocks to the marginal efficiency of investment, the equivalent of an investment wedge. They then point out that these investment shocks might proxy for financial frictions.

We show that with heterogeneity in investment technologies – as in much existing work – a credit crunch will show up as an investment wedge. Both Chari, Kehoe and McGrattan (2007) and Justiniano, Primiceri and Tambalotti (2011) take this mapping from financial frictions to wedges as given and examine whether it is borne out in the data. In this paper, we instead

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<sup>2</sup>In our framework, this corresponds to an extreme, binary, form of heterogeneous investment costs: either investment costs are zero, corresponding to the arrival of an investment opportunity, or infinite. For an example of continuous heterogeneity in investment technologies see Kurlat (2010).

<sup>3</sup>In Brunnermeier and Sannikov (2011) productivity is not stochastic and the two different productivity types are assumed to have different discount factors. The model therefore shares more similarity with our example in section 4.1.

<sup>4</sup>The idea of using such wedges to draw inferences about the sources of aggregate fluctuations goes back at least to Parkin (1988) who studies the labor wedge.

examine this mapping in more detail and point out that, depending on the form of heterogeneity, financial frictions can also manifest themselves as efficiency or labor wedges.

That financial frictions have the potential to cause aggregate productivity losses is a popular theme in the growth and development literature.<sup>5</sup> Buera, Kaboski and Shin (2010) and Moll (2010) also argue that aggregate capital accumulation – as measured by the capital-to-output ratio – is unaffected in their models with heterogeneous final goods producers. That a credit crunch can manifest itself as a labor wedge is the subject of Jermann and Quadrini (2009) and Arellano, Bai and Kehoe (2011).

On a more abstract level, our paper can be viewed as a cautious tale against drawing inferences about the sources of aggregate fluctuations from aggregate data alone. In this respect, it shares some similarity with the work by Chang and Kim (2007) and Chang, Kim and Schorfheide (2010) who examine a heterogeneous-agent economies with incomplete capital markets and indivisible labor. They show that a macroeconomist examining aggregate time-series generated by their model with neither distortions nor labor-supply shocks, would conclude that their economy features a time-varying labor wedge or preference shock, and that therefore abstracting from cross-sectional heterogeneity can potentially mislead policy predictions.

One of the main contributions of this paper is to derive analytic expressions for the various wedges despite the rich underlying heterogeneity. To deliver such tractability, we build on work by Angeletos (2007) and Kiyotaki and Moore (2008). Their insight is that heterogeneous agent economies remain tractable if individual production functions feature constant returns to scale because then individual policy rules are linear in individual wealth.<sup>6</sup>

Our paper is organized according to the different dimensions of heterogeneity we consider: heterogeneous productivity (Section 1), heterogeneous investment costs (Section 3), and heterogeneous recruitment costs (Section 3). In Section 4, we discuss some alternative modeling assumptions commonly used in the literature and their implications for our results. Section 5 is a conclusion.

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<sup>5</sup>Among others, see Banerjee and Duflo (2005), Jeong and Townsend (2007), Buera and Shin (2010), Buera, Kaboski and Shin (2010), Moll (2010).

<sup>6</sup>In contrast to the present paper, Angeletos focuses on the role of “uninsured idiosyncratic investment risk” and does not feature collateral constraints (except for the so-called “natural” borrowing constraint). Kiyotaki and Moore analyze a similar setup with borrowing constraints but their focus is on understanding the implications of monetary factors for aggregate fluctuations.

# 1 Benchmark Model: Heterogeneous Productivity

## 1.1 Preferences and Technology

Time is discrete. There is a continuum of entrepreneurs that are indexed by  $i \in [0, 1]$ . Entrepreneurs are heterogeneous in their productivity,  $z_{it}$ , their capital holdings,  $k_{it}$  and their debt,  $d_{it}$ . Each period, entrepreneurs draw a new productivity from a distribution  $\psi(z)$ . Importantly, this productivity shock is not only iid across entrepreneurs but also iid across time.<sup>7</sup> We assume a law of large numbers so the share of entrepreneurs experiencing any particular sequence of shocks is deterministic. Entrepreneurs have preferences

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_{it}), \quad u(c) = \frac{c^{1-\sigma}}{1-\sigma}. \quad (1)$$

Each entrepreneur owns a private firm which uses  $k_{it}$  units of capital and  $l_{it}$  units of labor to produce

$$y_{it} = f(z_{it}, k_{it}, l_{it}) = (z_{it}k_{it})^\alpha l_{it}^{1-\alpha} \quad (2)$$

units of output, where  $\alpha \in (0, 1)$ . Entrepreneurs also have access to the following linear technology to transform final goods into investment goods

$$k_{it+1} = x_{it} + (1 - \delta)k_{it}$$

where  $x_{it}$  is investment and  $\delta$  is the depreciation rate.

There is also a unit mass of workers. Workers have preferences over consumption and hours worked

$$\sum_{t=0}^{\infty} \beta^t [u(C_t^W) - v(L_t)] \quad (3)$$

where  $u$  is as in (1) and  $v$  is increasing and convex. For most of our results, we restrict the analysis to the case where workers do not have access to assets, and therefore, are hand-to-mouth consumers. We present numerical results for the case where workers have the same preferences as (3), can accumulate risk-free bonds, and face idiosyncratic labor endowment shocks.

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<sup>7</sup>In the appendix we analyze the case where productivity is persistent. The conclusions for the case of logarithmic utility function are unaffected by relaxing the assumption of iid shocks across time.

## 1.2 Budgets

Entrepreneurs hire workers in a competitive labor market at a wage  $w_t$ . They also trade in risk-free bonds. Denote by  $d_{it}$  the stock of bonds issued by an entrepreneur, that is his debt. When  $d_{it} < 0$  the entrepreneur is a net lender. The budget constraint is

$$c_{it} + x_{it} = y_{it} - w_t l_{it} - (1 + r_t) d_{it} + d_{it+1}. \quad (4)$$

Entrepreneurs face borrowing constraints

$$d_{it+1} \leq \theta_t k_{it+1}, \quad \theta_t \in [0, 1] \quad (5)$$

This formulation of capital market imperfections is analytically convenient. It says that at most a fraction  $\theta_t$  of next period's capital stock can be externally financed. Or alternatively, the down payment on debt used to finance capital has to be at least a fraction  $1 - \theta_t$  of the capital stock. The constraint can also be motivated as arising from a limited enforcement problem. Finally, note that by varying  $\theta_t$ , we can trace out all degrees of efficiency of capital markets;  $\theta_t = 1$  corresponds to a perfect capital market, and  $\theta_t = 0$  to the case where it is completely shut down. The implications of variations in  $\theta_t$  over the business cycle for aggregate GDP and capital are the main theme of this paper.

**Timing:** In order for there to be an interesting role for credit markets, an entrepreneur's productivity next period,  $z_{t+1}$ , is revealed at the end of period  $t$ , before the entrepreneur issues his debt  $d_{t+1}$ . That is, entrepreneurs can borrow to finance investment corresponding to their new productivity. Besides introducing a more interesting role for credit markets, a second purpose of this assumption is to eliminate "uninsured idiosyncratic investment risk". This is the focus of Angeletos (2007) and is well understood.

The budget constraint of entrepreneurs can be simplified slightly. The capital income of an entrepreneur is

$$\Pi(z_{it}, k_{it}, w_t) = \max_{l_{it}} (z_{it} k_{it})^\alpha l_{it}^{1-\alpha} - w_t l_{it} \quad (6)$$

Maximizing out over labor, we obtain the following simple and *linear* expression for profits:

$$\Pi(z_{it}, k_{it}, w_t) = z_{it} \pi_t k_{it}, \quad \pi_t = \alpha \left( \frac{1 - \alpha}{w_t} \right)^{(1-\alpha)/\alpha}. \quad (7)$$

This implies that the budget constraint of an entrepreneur reduces to

$$c_{it} + k_{it+1} = z_{it}\pi_t k_{it} + (1 - \delta)k_{it} - (1 + r_t)d_{it} + d_{it+1}. \quad (8)$$

We assume that workers cannot save so that they are in effect hand-to-mouth workers who immediately consume their earnings. Workers therefore only play a secondary role in the remainder of the analysis.<sup>8</sup>

### 1.3 Equilibrium

An *equilibrium* in this economy is defined in the usual way. That is, an equilibrium are sequences of prices  $\{r_t, w_t\}_{t=0}^{\infty}$ , and corresponding quantities such that (i) entrepreneurs maximize (1) subject to (8) and (5), taking as given  $\{r_t, w_t\}_{t=0}^{\infty}$ , and (ii) markets clear at all points in time:

$$\int d_{it} di = 0, \quad (9)$$

$$\int l_{it} di = L. \quad (10)$$

Summing up entrepreneurs' and workers' budget constraints and using these market clearing conditions, we also obtain the aggregate resource constraints of the economy which we find useful to state here.

$$C_t + X_t = Y_t, \quad K_{t+1} = X_t + (1 - \delta)K_t \quad (11)$$

$$C_t = C_t^E + C_t^W \quad (12)$$

Here,  $K_t, Y_t$  and  $X_t$  are the aggregate capital stock, output and investment.  $C_t$  is aggregate consumption which is the sum of total consumption by entrepreneurs,  $C_t^E$ , and workers,  $C_t^W$ .

### 1.4 Aggregate Wedges

The main goal of this paper is to study the mapping from a credit crunch to aggregate wedges. We follow the literature, in particular Chari, Kehoe and McGrattan (2007), and define these wedges as follows.

**Definition 1** Consider aggregate data  $\{K_t, L_t, Y_t, C_t\}_{t=0}^{\infty}$  generated by the model economy. The efficiency wedge is defined as  $A_t = Y_t K_t^{-\alpha} L_t^{-(1-\alpha)}$ . The labor wedge,  $\tau_{Lt}$ , is defined by the

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<sup>8</sup>We discuss in section 1.5.3 below how results change if we allow workers to save.

equation

$$\frac{v'(L_t)}{u'(C_t)} = (1 - \tau_{L_t})(1 - \alpha)\frac{Y_t}{L_t} \quad (13)$$

Finally, the investment wedge,  $\tau_{X_t}$ , is defined by the equation

$$u'(C_t)(1 + \tau_{X_t}) = \beta u'(C_{t+1}) \left[ \alpha \frac{Y_{t+1}}{K_{t+1}} + (1 - \delta)(1 + \tau_{X_{t+1}}) \right], \quad \text{all } t. \quad (14)$$

These wedges have the natural interpretation of productivity, and labor and investment taxes in a representative agent economy with resource constraint (11), Cobb-Douglas aggregate production function  $Y_t = A_t K_t^\alpha L_t^{1-\alpha}$  and preferences of the representative consumer given by  $\sum_{t=0}^{\infty} \beta^t [u(C_t) - v(L_t)]$ .

Equation (13) has the interpretation of the labor supply and labor demand conditions with the labor wedge corresponding to a labor income tax. Equation (14) has the interpretation of the Euler equation of the representative consumer and the investment wedge,  $\tau_{X_t}$ , then resembles a tax rate on investment.<sup>9</sup>

In our economy, by assumption only entrepreneurs invest; workers only supply labor. In answering the question whether aggregate investment is distorted, it will therefore sometimes be useful to examine what we term the *entrepreneurial investment wedge*. This object is analogous to the investment wedge just defined, but uses only aggregate data on quantities pertaining to entrepreneurs.

**Definition 2** Consider aggregate data  $\{K_t, Y_t, C_t^E\}_{t=0}^{\infty}$  generated by the model economy. The entrepreneurial investment wedge,  $\tau_{X_t}^E$ , is defined by the equation

$$u'(C_t^E)(1 + \tau_{X_t}^E) = \beta u'(C_{t+1}^E) \left[ \alpha \frac{Y_{t+1}}{K_{t+1}} + (1 - \delta)(1 + \tau_{X_{t+1}}^E) \right], \quad \text{all } t. \quad (15)$$

As we will show below, it turns out that the investment wedge,  $\tau_{X_t}$  does not necessarily equal the *entrepreneurial investment wedge*,  $\tau_{X_t}^E$ .<sup>10</sup>

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<sup>9</sup>More precisely, consider the following competitive equilibrium in this economy. The representative consumer maximizes his utility function subject to the budget constraint

$$C_t + (1 + \tau_{X_t})X_t = (1 - \tau_{L_t})w_t L + R_t K_t + T_t$$

and the capital accumulation law  $K_{t+1} = X_t + (1 - \delta)K_t$ , where  $R_t$  is the rental rate and  $T_t$  are lump-sum transfers. Equation (14) is the corresponding Euler equation. Further, a representative firm maximizes profits given by  $A_t K_t^\alpha L_t^{1-\alpha} - w_t L - R_t K_t$  so  $R_t = \alpha Y_t / K_t$  and  $w_t = (1 - \alpha)Y_t / L_t$ . Chari, Kehoe and McGrattan (2007) term this the “benchmark prototype economy”.

<sup>10</sup>It is easy to see that  $\tau_{X_t} \neq \tau_{X_t}^E$  if the marginal rate of substitution of the “representative worker”,  $u'(C_t^W)/[\beta u'(C_{t+1}^W)]$ , is different from that of the “representative entrepreneur”,  $u'(C_t^E)/[\beta u'(C_{t+1}^E)]$ . This

## 1.5 Log Utility

We find it instructive to first present our model and main result for the special case of log utility,  $\sigma = 1$ .

### 1.5.1 Individual Behavior

The problem of an entrepreneur can be written recursively as:

$$\begin{aligned} V_t(k, d, z_{-1}, z) &= \max_{c, d', k'} \log c + \beta \mathbb{E}[V_{t+1}(k', d', z, z')] \quad \text{s.t} \\ c + k' - d' &= z_{-1}\pi_t k + (1 - \delta)k - (1 + r_t)d, \quad d' \leq \theta_t k', \quad k' \geq 0. \end{aligned} \quad (16)$$

Here we denote by  $z_{-1}$  the productivity of an entrepreneur in the current period, by  $z$  his productivity in the next period, and by  $z'$  his productivity two periods ahead. The expectation is taken over  $z'$  only, because we assume that an entrepreneur knows  $z$  at the time he chooses capital and debt holdings. This problem can be simplified. To this end define an entrepreneur's "cash-on-hand",  $m_{it}$ , and "net worth",  $a_{it}$ , as

$$m_{it} \equiv z_{it}\pi_t k_{it} + (1 - \delta)k_{it} - (1 + r_t)d_{it}, \quad a_{it} \equiv k_{it} - d_{it} \quad (17)$$

**Lemma 1** *Using the definitions in (17), the following dynamic program is equivalent to (16):*

$$\begin{aligned} v_t(m, z) &= \max_{a'} \log(m - a') + \beta \mathbb{E}v_{t+1}(\tilde{m}_{t+1}(a', z), z') \\ \tilde{m}_{t+1}(a', z) &= \max_{k', d'} z\pi_{t+1}k' + (1 - \delta)k' - (1 + r_{t+1})d', \quad \text{s.t.} \\ k' - d' &= a', \quad k' \leq \lambda_t a', \quad \lambda_t \equiv \frac{1}{1 - \theta_t} \in [1, \infty) \end{aligned}$$

The interpretation of this result is that the problem of an entrepreneur can be solved as a two-stage budgeting problem. In the first stage, the entrepreneur chooses how much net worth,  $a'$ , to carry over to the next period. In the second stage, conditional on  $a'$ , he then solves an optimal portfolio allocation problem where he decides how to split his net worth between capital,  $k'$  and bonds,  $-d'$ . The borrowing constraint (5) immediately implies that the amount of capital he holds can be at most a multiple  $\lambda_t \equiv (1 - \theta_t)^{-1}$  of this net worth.  $\lambda_t$  is therefore the maximum attainable leverage. From now on, a credit crunch will interchangeably mean a drop in  $\theta_t$  or  $\lambda_t$ .

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is what will happen below.

**Lemma 2** *Capital and debt holdings are linear in net worth, and there is a productivity cutoff for being active  $\underline{z}_{t+1}$ .*

$$k_{it+1} = \begin{cases} \lambda_t a_{it+1}, & z_{it+1} \geq \underline{z}_{t+1} \\ 0, & z_{it+1} < \underline{z}_{t+1} \end{cases}, \quad d_{it+1} = \begin{cases} (\lambda_t - 1)a_{it+1}, & z_{it+1} \geq \underline{z}_{t+1} \\ -a_{it+1}, & z_{it+1} < \underline{z}_{t+1}. \end{cases} \quad (18)$$

The productivity cutoff is defined by

$$\underline{z}_{t+1} \pi_{t+1} = r_{t+1} + \delta.$$

Both the linearity and cutoff properties follow directly from the fact that individual technologies (2) display constant returns to scale in capital and labor. We have already shown that maximizing out over labor in (6), profits are linear in capital, (7). It follows that the optimal capital choice is at a corner: it is zero for entrepreneurs with low productivity, and the maximal amount allowed by the collateral constraints,  $\lambda_t a'$ , for those with high productivity. The productivity of the marginal entrepreneur is  $\underline{z}_{t+1}$ . For him, the return on one unit of capital  $z \pi_{t+1}$  equals the user cost of capital,  $r_{t+1} + \delta$ . The linearity of capital and debt delivers much of the tractability of our model.

**Lemma 3** *Entrepreneurs save a constant fraction of cash-on-hand:*

$$a_{it+1} = \beta m_{it+1}, \quad (19)$$

or using the definitions of cash-on-hand and net worth in (17)

$$k_{it+1} - d_{it+1} = \beta [z_{it} \pi_t k_{it} + (1 - \delta)k_{it} - (1 + r_t)d_{it}]. \quad (20)$$

### 1.5.2 Aggregation

Aggregating (20) over all entrepreneurs, we obtain our first main result:

**Proposition 1** *Aggregate quantities satisfy*

$$Y_t = Z_t K_t^\alpha L^{1-\alpha} \quad (21)$$

$$K_{t+1} = \beta [\alpha Y_t + (1 - \delta)K_t] \quad (22)$$

where

$$Z_t = \left( \frac{\int_{\underline{z}_t}^{\infty} z \psi(z) dz}{1 - \Psi(\underline{z}_t)} \right)^\alpha = \mathbb{E}[z | z \geq \underline{z}_t]^\alpha \quad (23)$$

is measured TFP. The cutoff is defined by

$$\lambda_{t-1}(1 - \Psi(\underline{z}_t)) = 1. \quad (24)$$

**Corollary 1** *Aggregate entrepreneurial consumption is given by  $C_t^E = (1 - \beta)[\alpha Y_t + (1 - \delta)K_t]$  and satisfies an Euler equation for the “representative entrepreneur”:*

$$\frac{C_{t+1}^E}{C_t^E} = \beta \left[ \alpha \frac{Y_{t+1}}{K_{t+1}} + 1 - \delta \right] \quad (25)$$

Aggregate consumption of workers is given by  $C_t^W = (1 - \alpha)Y_t$ .

### 1.5.3 A Credit Crunch

In this section, we conduct the following thought experiment: consider an economy that is in steady state at time,  $t = 0$ , with a given degree of financial friction,  $\lambda_0$  (equivalently,  $\theta_0$ ). At time  $t = 1$ , there is a credit crunch:  $\lambda_t$  falls and then recovers over time until it reaches the pre-crunch level of  $\lambda_0$ . We ask: what are the “impulse responses” of aggregate output, consumption and capital accumulation to this credit crunch?

**Proposition 2** *In our benchmark economy and under the assumption of log-utility, a credit crunch*

- (i) *is isomorphic to a drop in total factor productivity of magnitude defined by (23) and (24). Equivalently, a credit crunch does not distort the Euler equation of a “representative entrepreneur” which is given by*

$$\frac{C_{t+1}^E}{C_t^E} = \beta \left[ \alpha \frac{Y_{t+1}}{K_{t+1}} + 1 - \delta \right] \quad (26)$$

*and hence the entrepreneurial investment wedge is zero,  $\tau_{Xt}^E = 0$  for all  $t$ .*

- (ii) *results in an investment wedge,  $\tau_{Xt}$ , defined recursively by*

$$\frac{C_{t+1}}{C_t} \tau_{Xt} - \beta(1 - \delta)\tau_{Xt+1} = \frac{C_t^W}{C_t} \left[ \frac{C_{t+1}^E}{C_t^E} - \frac{C_{t+1}^W}{C_t^W} \right], \quad t \geq 1 \quad \tau_{X0} = 0. \quad (27)$$

A credit crunch distorts the investment decisions of individual entrepreneurs. One may have expected that therefore also the investment decision of a “representative entrepreneur” is distorted. Part (i) of the proposition states that this is not the case: a credit crunch lowers aggregate investment only to the extent that it lowers TFP and therefore the aggregate marginal product of capital; the wedge in the Euler equation of a representative entrepreneur is identically zero. This result is not straightforward. Much of the next subsection – which also covers the more general case of CRRA utility – will be concerned with discussing the intuition behind it. Part (ii) of the Proposition states that while aggregate investment is not distorted, there is nevertheless a non-zero investment wedge as in Definition 1. This is because, while the Euler equation of the “representative entrepreneur” is not distorted, the “representative worker” is borrowing constrained and has consumption  $C_t^W = (1 - \alpha)Y_t$ . Aggregate consumption is the sum of the consumption of workers and entrepreneurs. The aggregate investment wedge is found by matching up two equations: the growth rate of aggregate consumption and the equation defining the aggregate investment (14). It can easily be seen that a non-zero investment wedge is needed to match up these two equations. Its size depends on relative consumption growth of entrepreneurs and workers. We will argue momentarily that this investment wedge is “unimportant” in two respects: first, it is “small” and second it is “upside down”, in the sense of looking like a *subsidy* to investment as opposed to a tax. Furthermore, using simulations we show that the investment wedge is even smaller in the case where workers can save in a risk-less assets and face idiosyncratic labor income risk.

Figure 1 shows the effect of a credit crunch on aggregate TFP (panel a) and the investment wedge (panel b). Panel (a) simply restates

The determinants of the investment are further examined in panels (c) and (d).

However, note that the investment wedge is only non-zero during the transition to the new steady state; in steady state, when consumption is constant over time, the investment wedge is zero,  $\tau_{Xt} = 0$ .

## 1.6 General CRRA Utility

Consider now the case where individuals’ preferences are given by the general CRRA utility function (1). The analysis of the saving problem of individual entrepreneurs is similar to the log case analyzed in the preceding section.<sup>11</sup> We therefore relegate the details to Appendix B.

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<sup>11</sup>For  $\sigma \neq 1$ , the saving policy function cannot be solved in closed form anymore. While the saving policy function can still be shown to be linear in cash-on-hand, the saving rate now depends on future productivity,

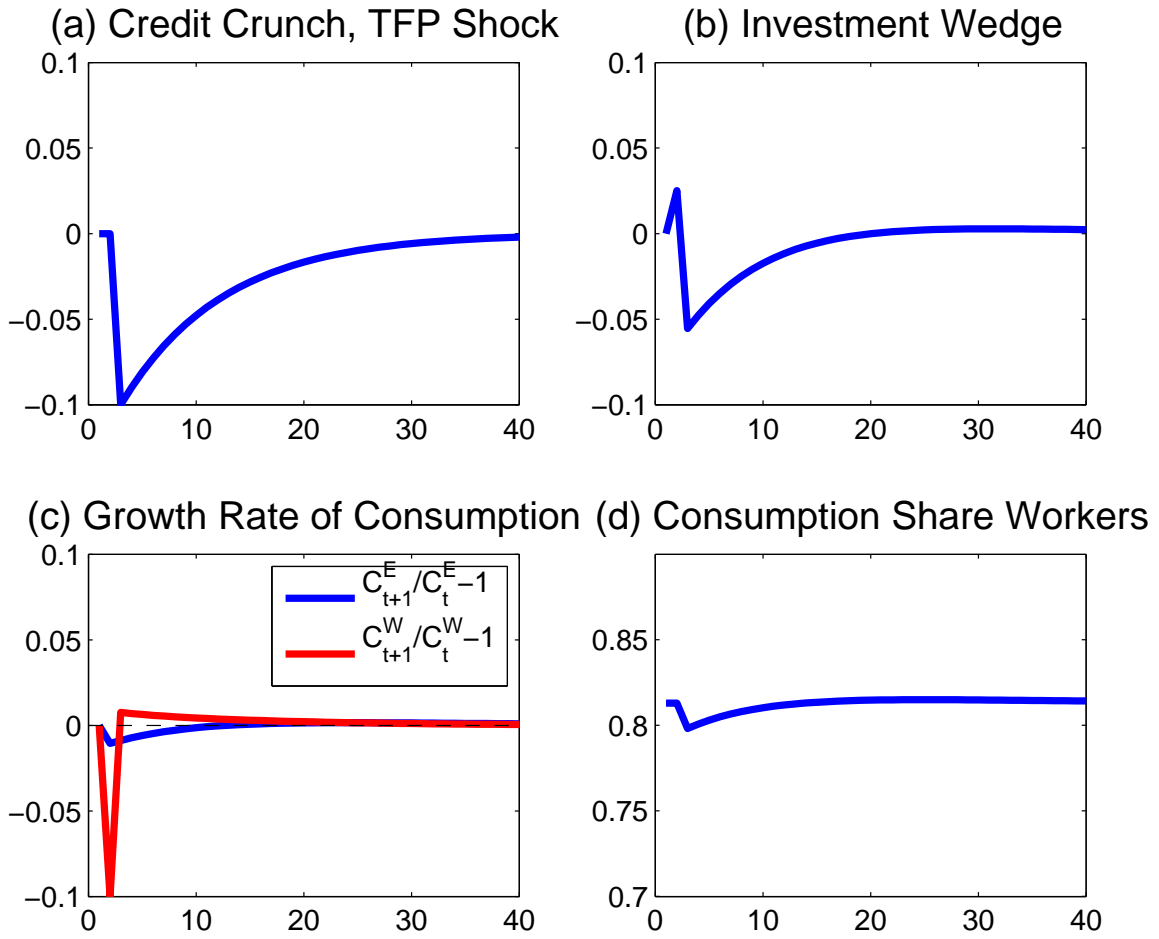


Figure 1: Response to a Credit Crunch

### 1.6.1 Individual Euler Equations

The Euler equation of an individual entrepreneur (with respect to net worth,  $a_{it+1}$ ) is

$$\frac{u'(c_{it})}{\beta \mathbb{E}[u'(c_{it+1})]} = R_{it+1}^a \quad (28)$$

where

$$R_{it+1}^a \equiv \max_{\phi \in [0, \lambda_t]} \{ \phi R_{it+1}^k + (1 - \phi)(1 + r_{t+1}) \} \quad (29)$$

is the return to wealth and

$$R_{it+1}^k \equiv \alpha \frac{y_{it+1}}{k_{it+1}} + 1 - \delta \quad (30)$$

is the return to capital. As already noted above, the savings decision involves an optimal portfolio allocation problem. Here,  $\phi$  is the share of net worth held in capital. If an entrepreneur's return to capital,  $R_{it+1}^k$  is higher than the return to bonds,  $r_{t+1}$ , he will take out as much debt as possible and invest his entire net worth into capital,  $\phi = \lambda_t \geq 1$ . Conversely, if the return to capital is less than the return to bonds, he holds his entire net worth in bonds,  $\phi = 0$ . Note that for credit constrained entrepreneurs (those with  $\phi = \lambda_t$ ) there is a positive wedge between their return to capital and their return to bonds,  $R_{it+1}^k - 1 - r_{t+1} > 0$ . Also, their return to savings is higher than the interest rate,  $R_{it+1}^a > 1 + r_{t+1}$ , which is to say that individual Euler equations are distorted.<sup>12</sup>

In contrast and as we have shown in Proposition 2 in the previous section, aggregate investment can be *undistorted* under certain conditions. The goal of this section is to show how to derive an Euler equation of the “representative entrepreneur” (of the form (26)) by directly aggregating the individual Euler equations (28). That is, we want to understand the conditions under which *distorted individual* Euler equations can be aggregated to obtain an *undistorted aggregate* Euler equation. The purpose of this exercise is mainly pedagogical. While the main result does not change, this exercise is not merely an alternative proof strategy. Instead, directly working with individual Euler equations allows for a much deeper understanding of the logic behind our result and underlines that this logic is, in fact, quite general.

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$z_{t+1}$  (which is known at time  $t$ ):  $a_{it+1} = s_{t+1}(z_{it+1})a_{it}$ . With log-utility  $s_{t+1}(z_{it+1}) = \beta$  is constant because the income and substitution effects of a higher productivity draw exactly offset each other.

<sup>12</sup>This is immediate because without financial frictions, all individual Euler equations would take the form

$$\frac{u'(c_{it})}{\beta \mathbb{E}[u'(c_{it+1})]} = 1 + r_{t+1}.$$

### 1.6.2 Euler Equation of Representative Entrepreneur

We aggregate (28) by taking a wealth weighted average to obtain:

$$\int \frac{u'(c_{it})}{\beta \mathbb{E}[u'(c_{it+1})]} \frac{a_{it+1}}{K_{t+1}} di = \int R_{it+1}^a \frac{a_{it+1}}{K_{t+1}} di \quad (31)$$

It is useful to separately analyze the left-hand-side and right-hand-side of this equation. We denote these by

$$\text{LHS} \equiv \int \frac{u'(c_{it})}{\beta \mathbb{E}[u'(c_{it+1})]} \frac{a_{it+1}}{K_{t+1}} di \quad \text{and} \quad (32)$$

$$\text{RHS} \equiv \int R_{it+1}^a \frac{a_{it+1}}{K_{t+1}} di. \quad (33)$$

Note that the left-hand-side encodes information about different entrepreneurs' marginal rates of substitution between consumption today and tomorrow, and the right-hand-side encodes information about the marginal rates of transformation, which are given by different entrepreneurs' returns to wealth.

### 1.6.3 Euler Equation of Representative Entrepreneur: Right-Hand Side

Consider first the right-hand side. By manipulating (33), we obtain the following Lemma whose proof is simple and therefore stated in the main text.

**Lemma 4 (RHS)** *A wealth weighted average of the return to wealth accumulation across entrepreneurs equals the aggregate marginal product of capital:*

$$\text{RHS} = \alpha \frac{Y_{t+1}}{K_{t+1}} + 1 - \delta$$

**Proof** The proof proceeds in two steps.

**Step 1:** Denote by  $\phi_{it+1} = k_{it+1}/a_{it+1}$ ,  $i$ 's optimal portfolio choice in (29) and combine with (33) to get

$$\text{RHS} = \int R_{it+1}^a \frac{a_{it+1}}{K_{t+1}} di = \int [\phi_{it+1} R_{it+1}^k + (1 - \phi_{it+1})(1 + r_{t+1})] \frac{a_{it+1}}{K_{t+1}} di \quad (34)$$

Next note that the bond market clearing condition (9) can be written as

$$0 = \int d_{it+1} di = - \int (1 - \phi_{it+1}) \frac{a_{it+1}}{K_{t+1}} di$$

Using this and  $\phi_{it+1} = k_{it+1}/a_{it+1}$ , (34) becomes

$$\text{RHS} = \int R_{it+1}^k \frac{k_{it+1}}{K_{t+1}} di.$$

**Step 2:** Using the definition of  $R_{it+1}^k$ , (30), we immediately get

$$\text{RHS} = \alpha \frac{Y_{t+1}}{K_{t+1}} + 1 - \delta. \square$$

Lemma 4 will be the main building block of the result that the Euler equation of a representative entrepreneur is not distorted (Proposition 3). The proof of the Lemma has two main steps: the first step is to show that a wealth weighted average of the returns to wealth equals a capital weighted average of the returns to capital:

$$\int R_{it+1}^a \frac{a_{it+1}}{K_{t+1}} di = \int R_{it+1}^k \frac{k_{it+1}}{K_{t+1}} di.$$

This result is remarkably general. In particular, it does *not* depend in any way on the form of utility or production functions. For example, the latter could display decreasing returns to scale. The intuition for this first step is not entirely straightforward and we spend some time discussing it in the next paragraph. The second step in the proof is to show that a capital weighted average of the returns to capital, (30), equals the aggregate marginal product of capital:

$$\int R_{it+1}^k \frac{k_{it+1}}{K_{t+1}} di = \alpha \frac{Y_{t+1}}{K_{t+1}} + 1 - \delta.$$

The assumption of Cobb-Douglas production functions is crucial for this step because it implies that the marginal product of capital is proportional to the average product. Given the Cobb-Douglas assumption, this second step is relatively mechanical and we will not discuss it further.

The key to understanding the first step of the Lemma, is a general equilibrium effect: the interest rate adjusts in such a way that all effects of financial frictions get absorbed into the average return to capital. To gain some intuition, consider an economy that starts in equilibrium with  $(\lambda_t, r_{t+1}) = (\lambda_0, r_0)$ . At time  $t$ , a credit crunch hits and leverage decreases to  $\lambda_1 < \lambda_0$ . In order to trace out the economy's response, we find it useful to index variables by  $(\lambda, r)$ . For instance, we write  $k_i(\lambda, r)$  for the capital demand of entrepreneur  $i$  when leverage is  $\lambda$  and the interest rate is  $r$ . We also suppress time subscripts for notational simplicity.

Consider first, the response of this economy in partial equilibrium, that is when the interest rate is fixed at  $r = r_0$ . An immediate effect of the credit crunch is that credit is restricted:

in partial equilibrium, aggregate capital demand is now smaller than aggregate capital supply and given by

$$K^D(\lambda_1, r_0) = \int k_i(\lambda_1, r_0) di = \lambda_1 (1 - \Psi(\underline{z}(r_0))) K < K \quad (35)$$

Following similar steps as in Lemma 4, the wealth weighted average of individual returns to wealth can easily be shown to be given by

$$\int R_i^a(\lambda_1, r_0) \frac{a_i}{K} di = \int R_i^k \frac{k_i(\lambda_1, r_0)}{K^D(\lambda_1, r_0)} di \frac{K^D(\lambda_1, r_0)}{K} + (1 + r_0) \left( 1 - \frac{K^D(\lambda_1, r_0)}{K} \right).$$

It follows immediately from the fact that aggregate capital demand falls in partial equilibrium, (35), that

$$\int R_i^a(\lambda_1, r_0) \frac{a_i}{K} di < \int R_i^k \frac{k_i(\lambda_1, r_0)}{K^D(\lambda_1, r_0)} di. \quad (36)$$

In partial equilibrium, a credit crunch results in the average return to wealth to *fall below* the average return to capital. This is because aggregate leverage  $K^D(\lambda_1, r_0)/K$  falls. The implication is that in partial equilibrium, a credit crunch looks like the introduction of a tax on the returns to capital, with the right-hand side of (36) corresponding to the pre-tax return and the left-hand side to the after-tax return. Put another way: in partial equilibrium, the entrepreneurial investment wedge is positive.

In general equilibrium, however, the implication of a credit crunch is quite different. An immediate implication of (35) is that the interest rate must fall until  $K^D(\lambda_1, r_1) = K$ . This immediately implies that

$$\int R_i^a(\lambda_1, r_1) \frac{a_i}{K} di = \int R_i^k \frac{k_i(\lambda_1, r_1)}{K} di$$

In general equilibrium, aggregate leverage increases to the pre-credit crunch level, and hence the average return to wealth again equals the average return to capital.

This general equilibrium effect obviously hinges on our assumption of a closed economy. In an open economy a credit crunch would lead to an entrepreneurial investment wedge. Another crucial assumption is that the borrowing constraint takes the form (5). To gain some further intuition, it is useful to examine where the proof breaks down if this assumption is not satisfied. Consider a more general borrowing constraint of the form  $k_{it+1} \leq b_{it+1}(a_{it+1}, z_{it+1}, r_{t+1}, w_{t+1}, \dots)$ . In this case, the return to wealth is

$$R_{it+1}^a = 1 + r_{t+1} + \frac{\partial b_{it+1}}{\partial a_{it+1}} [R_{it+1}^k - 1 - r_{t+1}]$$

which can be shown to imply that

$$\text{RHS} = \int R_{it+1}^k \frac{\partial \log b_{it+1}}{\partial \log a_{it+1}} \frac{k_{it+1}}{K_{t+1}} di + (1 + r_{t+1}) \int \left(1 - \frac{\partial \log b_{it+1}}{\partial \log a_{it+1}}\right) \frac{k_{it+1}}{K_{t+1}} di. \quad (37)$$

It can be seen that the feature of the borrowing constraint that is critical is that its elasticity with respect to wealth is one, meaning that it takes the form  $k_{it+1} \leq \lambda(z_{it+1}, r_{t+1}, w_{t+1} \dots) a_{it+1}$ . Apart from that it can be a general function of other attributes of a given type of entrepreneur  $i$ , e.g., their productivity, and prices.

#### 1.6.4 Euler Equation of Representative Entrepreneur: Left-Hand Side

Consider next the left-hand side of (31). By manipulating (32), we obtain the following Lemma.

##### Lemma 5 (LHS)

$$\text{LHS} = \frac{C_{t+1}^E}{C_t^E} \frac{1}{\bar{s}_{t+1}} \frac{1 - \bar{s}_{t+1}}{1 - \bar{s}_{t+2}} \quad \text{where} \quad \bar{s}_{t+1} = \int_0^\infty s_{t+1}(z) \psi(z) dz \quad (38)$$

and  $s_{t+1}(z)$  is the saving rate of type  $z$ .

For the special case of log-utility,  $\sigma = 1$ , all entrepreneurs save the same fraction of their cash-on-hand regardless of their type,  $s_t(z) = \beta$ . Hence (38) specializes to

$$\text{LHS} = \frac{C_{t+1}^E}{\beta C_t^E}$$

Together with Lemma 4, the assumption of log-utility then implies the Euler equation for the “representative entrepreneur” in (26).<sup>13</sup>

#### 1.6.5 A Credit Crunch

More generally, we can obtain the following characterization of the distortions implied by a credit crunch.

**Proposition 3** *In our benchmark economy, a credit crunch*

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<sup>13</sup>Similarly, the law of motion of the aggregate capital stock in the economy with CRRA utility is

$$K_{t+1} = \bar{s}_{t+1} [\alpha Y_t + (1 - \delta) K_t], \quad \bar{s}_{t+1} = \int_0^\infty s_{t+1}(z) \psi(z) dz$$

For the special case  $\sigma = 1$ , and hence  $s_t(z) = \beta$ , we obtain (22).

(i) results in an entrepreneurial investment wedge

$$\frac{1}{\beta} \left( \frac{C_{t+1}^E}{C_t^E} \right)^\sigma (1 + \tau_{Xt}^E) - (1 - \delta) \tau_{Xt+1}^E = \frac{C_{t+1}^E}{C_t^E} \frac{1}{\bar{s}_{t+1}} \frac{1 - \bar{s}_{t+1}}{1 - \bar{s}_{t+2}} \quad (39)$$

where the initial (steady state) entrepreneurial investment wedge is

$$\tau_{X0}^E = \frac{\beta/\bar{s} - 1}{1 - \beta(1 - \delta)}$$

(ii) results in an investment wedge,  $\tau_{Xt}$ , defined recursively by

$$\left[ \frac{C_{t+1}^E}{C_t^E} + \frac{C_t^W}{C_t} \left( \frac{C_{t+1}^W}{C_t^W} - \frac{C_{t+1}^E}{C_t^E} \right) \right]^\sigma (1 + \tau_{Xt}) - \left( \frac{C_{t+1}^E}{C_t^E} \right)^\sigma (1 + \tau_{Xt}^E) = \beta(1 - \delta)(\tau_{Xt+1} - \tau_{Xt+1}^E), \quad t \geq 1, \quad (40)$$

where the initial (steady state) investment wedge is  $\tau_{X0} = \tau_{X0}^E$ .

Note that, for the case of log-utility  $\sigma = 1$ , the entrepreneurial investment wedge  $\tau_{Xt}^E = 0$ . This is because in that case  $\bar{s}_t = \beta$ .

Figure 2 shows the effect of a credit crunch on TFP and the entrepreneurial and aggregate investment wedges for three different values of the inverse of the intertemporal elasticity of substitution,  $\sigma$ . A value of  $\sigma = 1$  corresponds to log-utility and therefore the transition dynamics for that case are identical to Figure 1. Two other insights emerge from the Figure. First, the transition dynamics for the cases  $\sigma = 0.5$  and  $\sigma = 1.5$  are qualitatively very similar to those for the case  $\sigma = 1$ . That is, while the entrepreneurial investment wedge,  $\tau_{Xt}^E$ , is non-zero for the case  $\sigma \neq 1$ , it is small (panel b). As a consequence, the evolution of the aggregate investment wedge,  $\tau_{Xt}$ , is very similar to the log-utility case (panel c). Second, the entrepreneurial investment wedge is positive for the case where  $\sigma < 1$  and negative for the case  $\sigma > 1$ . This is intuitive: if entrepreneurs are relatively unwilling to substitute intertemporally ( $\sigma$  is high), they overaccumulate assets. In aggregate data, this looks like a subsidy to savings.

## 2 Heterogeneous Investment Costs

We have argued in the previous two sections that in an economy with heterogeneity in productivity, a credit crunch shows up in TFP; in contrast, the investment wedge is either zero or small. The purpose of the next two sections is to argue that this is by no means necessarily the case. If heterogeneity takes a different form, a credit crunch can show up as either an

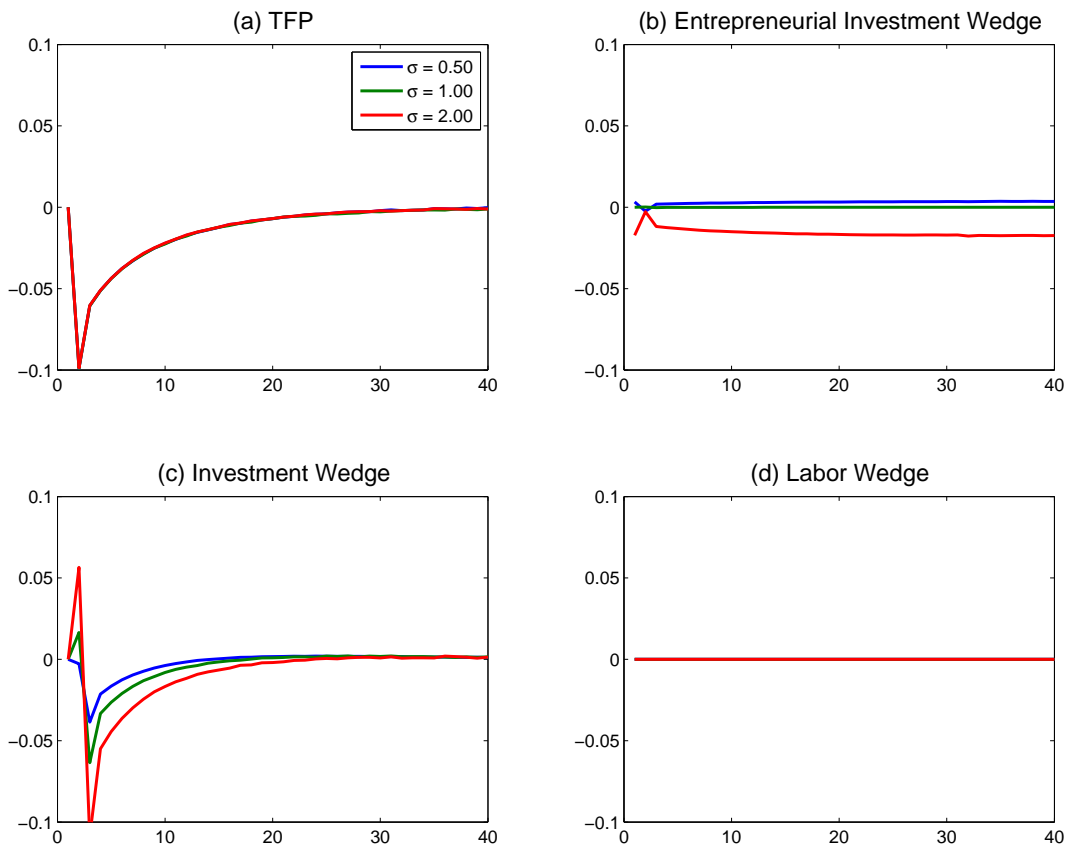


Figure 2: Response to a Credit Crunch: General CRRA Utility

investment or a labor wedge. In this section, we consider the case of heterogeneous investment costs and show that a credit crunch manifests itself as an investment wedge while aggregate TFP is unaffected by construction.

The economy is essentially the same as in the section 1 but differs in one important aspect: we replace heterogeneity in the productivity of final goods producers with heterogeneity in investment costs. To obtain one unit of investment goods, different entrepreneurs have to give up different amounts of consumption goods. The role of credit markets is then to reallocate funds towards those entrepreneurs with low investment costs.

Besides allowing us to make the point that different forms of heterogeneity have different aggregate implications, the case of heterogeneous adjustment is also useful to relate to much of the existing literature on financial frictions and business cycles. In particular, a number of papers make the assumption that each period “investment opportunities” arrive randomly to some exogenous fraction of entrepreneurs. Only entrepreneurs with an “investment opportunity” can acquire new investment goods; others cannot.<sup>14</sup> In our framework, this corresponds to an extreme form of heterogeneous investment costs: either investment costs are zero, corresponding to the arrival of an investment opportunity, or infinite.

## 2.1 Preferences, Technology and Budgets

There is a representative final goods producer with technology

$$Y_t = A_t K_t^\alpha L^{1-\alpha}$$

Hence there is no heterogeneity in final goods production.<sup>15</sup> Since TFP is exogenous, an immediate implication is that a credit crunch cannot result in an efficiency wedge by assumption.

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<sup>14</sup>The following papers all feature such heterogeneous “investment opportunities”: Bernanke and Gertler (1989), Carlstrom and Fuerst (1997) Kiyotaki and Moore (1997), Bernanke, Gertler and Gilchrist (1998), Kiyotaki and Moore (2005), Kiyotaki and Moore (2008), Gertler and Kiyotaki (2010), and Kurlat (2010). Exceptions with heterogeneous productivity are Kiyotaki (1998), Brunnermeier and Sannikov (2011), and Khan and Thomas (2010).

<sup>15</sup>An alternative assumption that also implies that final goods production can be summarized by an aggregate production function is that there is heterogeneity in productivity but final goods producers don’t face any credit (or other) constraints. In that case, marginal products of capital and labor are equalized. Hence total production solves

$$F(K, L) = \max_{k_i, l_i} \int (z_i k_i)^\alpha l_i^{1-\alpha} di, \quad \int k_i di \leq K, \quad \int l_i di.$$

and it can be shown that aggregate production takes the form  $F(K, L) = AK^\alpha L^{1-\alpha}$ . The fact that homogeneity of final goods producers is equivalent to perfect credit markets for final goods producers underlines again that the important feature of a model is how credit constraints *interact* with heterogeneity.

Final goods producers rent capital from entrepreneurs at a rental rate  $R_t$ . In equilibrium,

$$R_t = \alpha \frac{Y_t}{K_t}.$$

There is still a continuum of entrepreneurs indexed by  $i \in [0, 1]$ . These entrepreneurs have the same preferences as before, (1), but to make our point in the simplest way, we restrict the analysis to the case of log-utility  $\sigma = 1$ . They own and accumulate capital, and rent it to the representative firm. Entrepreneurs differ in their investment costs which we denote by  $\omega_{it}$ . To increase the capital stock by  $x_{it}$  units of capital, an entrepreneur has to give up  $\omega_{it}x_{it}$  units of the final good where  $\omega_{it} \geq 1$ . Each period, entrepreneurs draw a new investment cost from a distribution  $\psi(\omega)$ . The budget constraint of an entrepreneur is therefore

$$c_{it} + \omega_{it}x_{it} = R_t k_{it} - (1 + r_t)d_{it} + d_{it+1}$$

The law of motion for capital and the borrowing constraint are unchanged and given by (20) and (5). As before, entrepreneurs simply maximize their utility subject to these constraints. We also continue to assume that workers don't save and simply consume their labor income.

## 2.2 Aggregation and Credit Crunch

To answer the question whether there will be an investment wedge in this economy, we can aggregate individual Euler equations in a similar fashion to Lemmas 4 and 5.

**Proposition 4** *In the economy with heterogenous adjustment costs, the Euler equation of the “representative entrepreneur” takes the form*

$$\frac{C_{t+1}^E}{\beta C_t^E} \int \omega_{it} \frac{k_{it+1}}{K_{t+1}} di = \alpha \frac{Y_{t+1}}{K_{t+1}} + (1 - \delta) \int \omega_{it+1} \frac{k_{it+1}}{K_{t+1}} di \quad (41)$$

Therefore, a credit crunch results in an entrepreneurial investment wedge,  $\tau_{X_t}^E$ , defined recursively by

$$\frac{C_{t+1}^E}{\beta C_t^E} \tau_{X_t}^E - (1 - \delta) \tau_{X_{t+1}}^E = \frac{C_{t+1}^E}{\beta C_t^E} \int \omega_{it} \frac{k_{it+1}}{K_{t+1}} di - (1 - \delta) \int \omega_{it+1} \frac{k_{it+1}}{K_{t+1}} di \quad (42)$$

In contrast to the case with heterogenous productivity, heterogeneous investment costs imply that the Euler equation of a representative entrepreneur (41) is distorted. With imperfect credit markets, some entrepreneurs with investment costs,  $\omega_{it} > 1$  will be active and hold positive

capital stocks,  $k_{it+1} > 0$  and therefore

$$\int \omega_{it} \frac{k_{it+1}}{K_{t+1}} di > 1, \quad \int \omega_{it+1} \frac{k_{it+1}}{K_{t+1}} di > 1.$$

Comparing this aggregate Euler equation to the equation defining the entrepreneurial investment wedge, (14), it is obvious that  $\tau_{X_t}^E \neq 0$ . The second part of the proposition makes this intuition precise. It is in fact tempting to set the entrepreneurial investment wedge equal to  $\tau_{X_t}^E = \int \omega_{it} (k_{it}/K_t) di$ . However, this would be incorrect because the weights on  $\omega_{it}$  are given by  $k_{it+1}/K_{t+1}$  rather than  $k_{it}/K_t$ . Hence the more complicated definition of  $\tau_{X_t}^E$  in (42) is needed.

Summarizing, in a model with heterogeneous investment costs the results from the model with heterogeneous productivities are reversed: a credit crunch results in an entrepreneurial investment wedge and – by construction – in no efficiency wedge. This is illustrated in Figure 3.

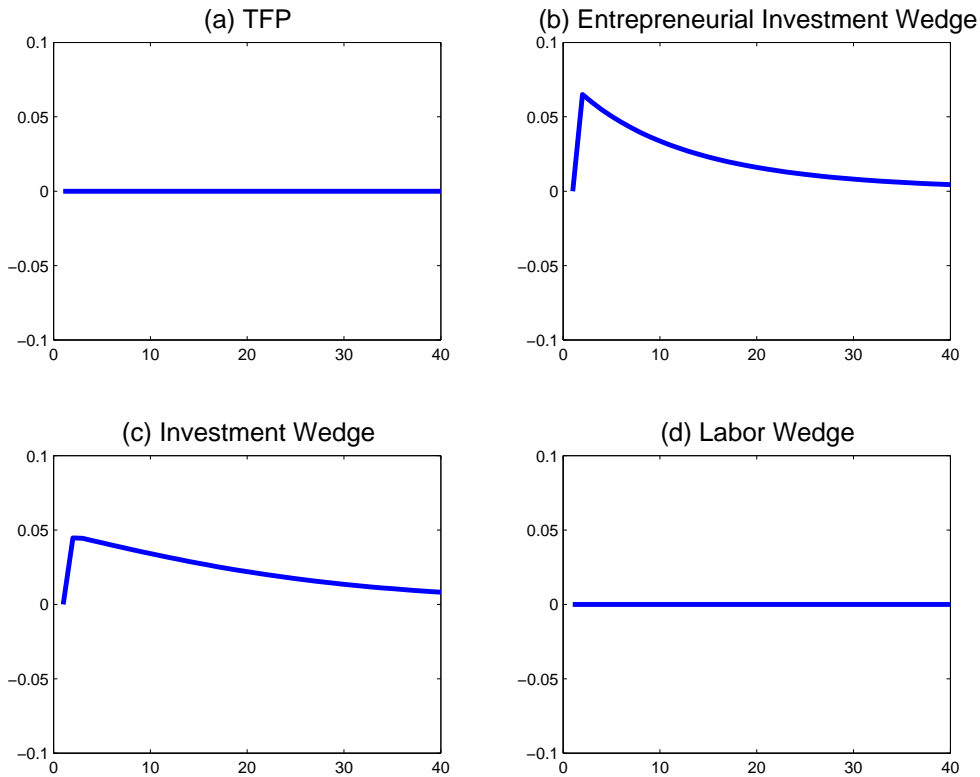


Figure 3: Response to a Credit Crunch: Heterogeneous Investment Costs

### 3 Heterogenous Recruitment Costs

We have shown that two different assumptions on the dimension along which individual entrepreneurs are heterogenous can lead to a credit crunch resulting in either an efficiency or an investment wedge. In this section, we show that with heterogeneity in yet another dimension, namely labor recruitment costs, a credit crunch can also show up as a labor wedge.

Our starting point is the observation that with some form of labor search frictions, labor looks very much like capital. In particular, search models typically have the feature that, in order to increase their labor force, firms have to post vacancies one period in advance, exactly in the same way as they have to invest to increase their stock of physical capital.<sup>16</sup> The fact that firms have to choose their labor force one period in advance, implies that financial frictions have the potential to affect employment and hence the labor wedge.<sup>17</sup>

We show in this section that an extension of our previous model that features labor search frictions, in combination with heterogeneity across entrepreneurs in the cost of recruiting, can indeed deliver a labor wedge. The result follows exactly the same logic as our previous results on the investment and efficiency wedges. In particular, note that in order for a credit crunch to show up as a labor wedge, the heterogeneity has to be chosen in “the right way.” Here, a credit crunch affects the allocation of labor across entrepreneurs with different recruitment costs in such a way that the aggregate cost of recruiting increases which delivers a drop in employment and hence an increase in the labor wedge. If instead, our model were to feature heterogeneity in productivity, a credit crunch would show up as a TFP wedge only.

It should also be noted that heterogenous recruitment costs are not merely a theoretical construct that we use to make our point. For instance, Davis, Faberman and Haltiwanger (2010) examine US data and find substantial heterogeneity in the cross-section of the “vacancy yield” of firms (the number of realized hires per reported job opening).

#### 3.1 Preferences, Technology and Budgets

There is again a continuum of entrepreneurs indexed by  $i \in [0, 1]$ . They have the preferences in (1). Each entrepreneur employs  $l_{it}$  and produces

$$y_{it} = A_t l_{it}.$$

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<sup>16</sup>For a formulation where this is very apparent see Shimer (2010).

<sup>17</sup>For other frameworks in which financial frictions result in a labor wedge, see Jermann and Quadrini (2009), Arellano, Bai and Kehoe (2011).

units of output. Note that, in contrast to the previous sections, there is no capital for simplicity. Also productivity,  $A_t$  is homogenous across firms. Therefore there is no efficiency wedge by assumption.

In Appendix C we work out the model where entrepreneurs are instead heterogenous in their productivity. We show that in this model, as already noted, a credit crunch results in an efficiency wedge. An entrepreneur's employment evolves according to

$$l_{it+1} = x_{it} + (1 - \delta)l_{it}, \quad (43)$$

where  $x_{it}$  is the number of new hires and  $\delta$  is the exogenous rate of job separations. In order to hire a worker, an entrepreneur has to post a costly vacancy. We assume that in order to attract  $x_{it}$  workers, an entrepreneur has to post  $\omega_{it}x_{it}$  vacancies. We refer to  $1/\omega_{it}$  as the "vacancy yield".  $\omega_{it}$  is drawn from  $\psi(\omega)$ , and is assumed to be iid across entrepreneurs and over time.

Posting one vacancy costs one unit of the consumption good and hence the budget constraint of an entrepreneur is

$$c_{it} + \omega_{it}x_{it} - d_{it+1} = A_t l_{it} - w_t l_{it} - (1 + r_t)d_{it} \quad (44)$$

Note that we assume that all entrepreneurs pay a common wage,  $w_t$ . Given that search frictions introduce the possibility of different wage determination mechanisms and that these search frictions are heterogenous across firms, this is not necessarily the case. However, we show below that such a common wage is consistent with individual rationality. We therefore proceed using the assumption of a common wage.

Since there is no capital in this economy, we change our borrowing constraint slightly. We assume that an entrepreneur can issue debt worth at most a fraction  $\theta_t$  of output in the next period:<sup>18</sup>

$$d_{it+1} \leq \theta_t A_t l_{it+1}. \quad (45)$$

Entrepreneurs then maximize their utility, (1), subject to (43), (44) and (45).

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<sup>18</sup>This can again be motivated with a limited commitment problem: entrepreneurs can default on their loans. In this case, a creditor can obtain a fraction  $\theta_t$  of output  $y_{it}$ . Knowing this, the creditor restricts his loan to be less than  $\theta_t y_{it}$ .

Workers have preferences (3) which we specialize to<sup>19</sup>

$$\sum_{t=0}^{\infty} \beta^t [u(C_t^W) - v(L_t)], \quad u(C) = \log C, \quad v(L) = \gamma L \quad (46)$$

We continue to assume that workers cannot save and simply consume their labor income,  $C_t^W = w_t L_t$ .

With the preferences in (46), the marginal rate of substitution between leisure and consumption is given by

$$\frac{v'(L)}{u'(C)} = \gamma C \quad (47)$$

Using that in our economy without capital,  $\alpha = 0$  and  $C_t = Y_t$ , the labor wedge – as defined in (13) – reduces to

$$\tau_{L_t} = 1 - \gamma L_t. \quad (48)$$

From (48) it is immediately apparent that any fluctuations in the labor wedge must be due to fluctuations in employment. This is because with the preferences in (3), the labor supply curve is vertical and given by  $1/\gamma$ . In the absence of search frictions, labor supply would equal labor demand and the labor wedge would be identically zero. Because search frictions provide a rationale for unemployment, labor demand and supply are no longer equalized giving rise to the possibility of a labor wedge that fluctuates over the business cycle.

### 3.2 Wages

In models with search frictions, wages are typically determined through Nash-bargaining between employers and employees. We work out the Nash bargaining solution in Appendix (A.9.2) and show that the fact that entrepreneurs are heterogenous in their recruitment costs,  $\omega_{it}$ , results in entrepreneur-specific wages being paid. This makes the Nash solution somewhat complicated to work with, in particular given that our stated goal is to derive simple characterizations of aggregate variables. We therefore pursue a different approach in the main text, exploiting the well-known fact that search models typically feature a *set* of wages that

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<sup>19</sup>As in Shimer (2010), this utility function can be interpreted as arising from a problem with indivisible labor. Consider a household with a continuum of members indexed by  $j \in [0, 1]$ . Each household member may be either employed by an entrepreneur,  $n_{jt} = 1$ , or unemployed,  $n_{jt} = 0$ . A member's period utility is  $\log c_{jt}$  if unemployed and  $\log c_{jt} - \gamma$  if employed. The household allocates total consumption in period  $t$ ,  $C_t^W$ , in order to maximize the sum of household utility, and so equalizes the marginal utility of consumption across individuals. With additive separability between consumption and leisure, this implies the household equalizes consumption across individuals, acting as if it has the utility function (3).  $L_t$  is then the fraction of household members who are employed in period  $t$ .

workers are willing to accept and that employers are willing to pay (Hall, 2005). Any such wage satisfies the condition that no worker-employer pair has an unexploited opportunity for mutual improvement. This is useful because there is, in particular, a *common* wage that is in this bargaining set.

**Lemma 6** *A sufficient condition for a common wage,  $w_t$ , to be in the bargaining set is*

$$\gamma C_t^W \leq w_t \leq A_t$$

This Lemma simply states that any wage greater than the marginal rate of substitution,  $\gamma C_t^W$ , but smaller than the marginal product of labor,  $A_t$ , is in the bargaining set.<sup>20</sup> We then simply impose an ad-hoc wage rule, namely that the wage always lies exactly halfway between the bounds in Lemma (6):

$$w_t = \frac{\gamma C_t^W + A_t}{2}$$

Since workers are hand-to-mouth workers,  $C_t^W = w_t L_t$ , we immediately get that the common wage is

$$w_t = \frac{A_t}{2 - \gamma L_t}.$$

### 3.3 Individual Behavior

We obtain the following characterization of an entrepreneur’s optimal choice of recruiters and hence workers next period.

**Lemma 7** *The optimal labor choice of an entrepreneur satisfies*

$$\omega_{it} l_{it+1} - d_{it+1} = \beta [A l_{it} (1 + (1 - \delta)\omega_{it}) - w_t l_{it} - (1 + r)d_{it}] \quad (49)$$

Note that this expression is of the same form as the optimal savings policy function in the case with debt-constrained capital accumulation, (20). The term in brackets on the right-hand-side of (49) is an entrepreneur’s “cash-on-hand”. The assumption of log-utility then implies that he then “saves” a constant fraction  $\beta$  of this “cash-on-hand”. Here, one of the entrepreneur’s assets is his stock of workers, valued by their opportunity cost in terms of final goods,  $\omega_{it} l_{it+1}$ .

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<sup>20</sup>The same condition is made use of in Blanchard and Gali (2010).

### 3.4 Aggregation and Credit Crunch

We want to show that in the present model with heterogenous recruitment costs, a credit crunch results in a labor wedge. To do so, we aggregate (49) over all entrepreneurs and obtain the following characterization of the employment and hence the labor wedge.

**Proposition 5** *Aggregate employment evolves according to*

$$L_{t+1} = \beta \Omega_t^{-1} \left[ A_t + (1 - \delta) \int \omega_{it} \frac{l_{it}}{L_t} di - w_t \right] L_t, \quad w_t = \frac{A_t}{2 - \gamma L_t}$$

where

$$\Omega_t \equiv \int \omega_{it} \frac{l_{it+1}}{L_{t+1}} di$$

is the “aggregate recruitment cost”. A credit crunch increases  $\Omega_t$  and hence decreases employment,  $L_{t+1}$ , resulting in an increase of the labor wedge,  $\tau_{L_{t+1}}$ .

Figure 4 graphically illustrates the response to a credit crunch in the economy with heterogenous recruitment costs.

## 4 Discussion of Alternative Modeling Assumptions

### 4.1 Heterogenous Discount or Death Rates.

A common assumption in the literature on financial frictions is that borrowers and lenders have heterogenous discount rates, or die at differential rates.<sup>21</sup> In the absence of idiosyncratic shocks, say to productivity as above, the purpose of this assumption is to ensure that financial frictions have some bite in the long-run. Without this assumption constrained borrowers would save themselves out of the borrowing constraint rendering any such constraints irrelevant in the long-run. We argue in this section that these assumptions tend to result in an investment wedge because they mechanically distort individual savings. We concentrate on the case of heterogenous discount rates but note that heterogenous death rates function in much the same way.

Consider a simplification of the above model with only two types of entrepreneurs: borrowers and lenders, respectively indexed by  $B$  and  $L$ . Workers are assumed to be hand-to-mouth

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<sup>21</sup>For an example of the former assumption, see Carlstrom and Fuerst (1997). For the latter see Bernanke, Gertler and Gilchrist (1998) and Gertler and Karadi (2011).

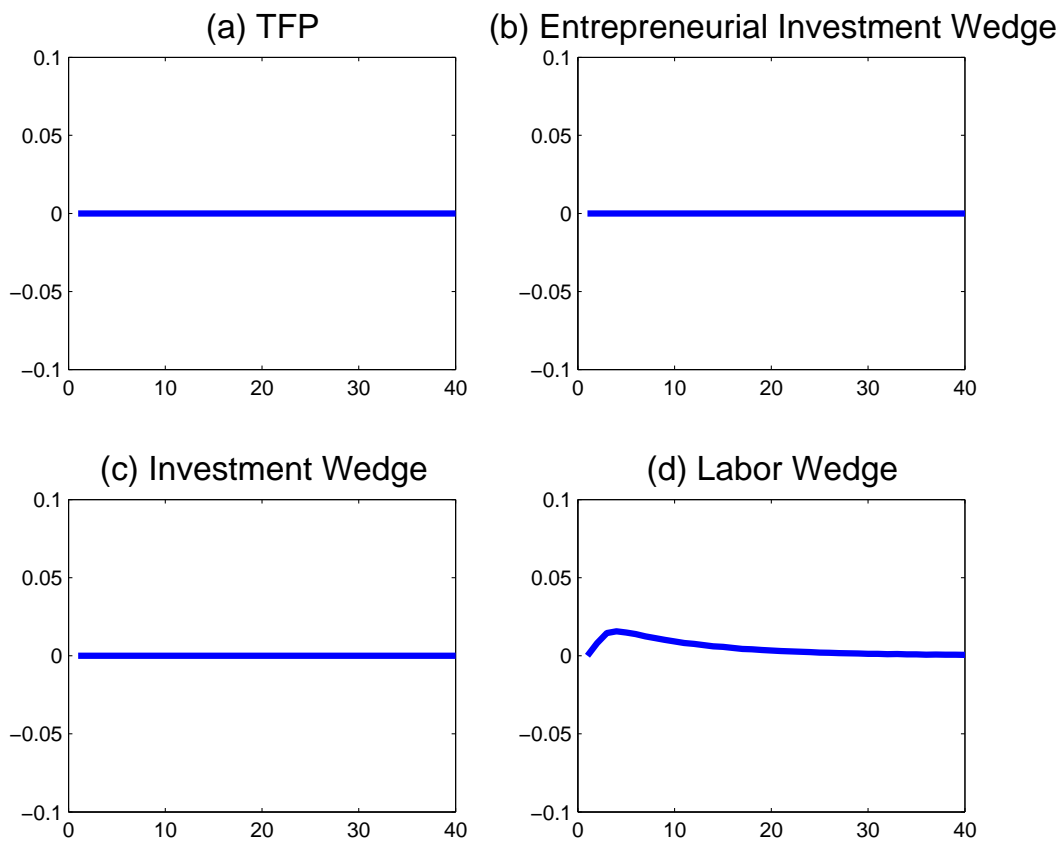


Figure 4: Response to a Credit Crunch: Heterogenous Recruitment Costs

workers as before. Both borrowers and lenders have preferences (1) with  $\sigma = 1$ . Only borrowers have access to a production technology  $Y_t = AK_t^\alpha L_t^{1-\alpha}$ . Hence borrowers produce all output, own all capital and employ all labor in this economy and – as the name already suggests – borrow in order to invest and accumulate capital. Their productivity,  $A$ , is assumed to be fixed. Importantly, borrowers and lenders differ in their discount factors:

$$\beta_B < \beta_L$$

Borrowers and lenders face the following budget constraints

$$\begin{aligned} c_{Bt} + K_{t+1} - d_{Bt+1} &= Y_t - w_t L_t + (1 - \delta)K_t - (1 + r_t)d_{Bt} \\ c_{Lt} - d_{Lt+1} &= -(1 + r_t)d_{Lt} \end{aligned}$$

Clearly, the model so far is the special case of the model in section 1 with only two types of entrepreneurs,  $i \in \{B, L\}$ , that have productivities  $(z_B, z_L) = (A^{1/\alpha}, 0)$ . We depart slightly from the model above and assume that the borrowing constraint faced by entrepreneurs takes the form

$$(1 + r_{t+1})d_{Bt+1} \leq \theta_t[Y_{t+1} - w_{t+1}L_{t+1} + (1 - \delta)K_{t+1}] \quad (50)$$

The value of debt taken out by borrowers has to be collateralized by at least a fraction  $\theta_t \in [0, 1]$  of next period's profits plus undepreciated capital. This constraint is slightly more convenient than (5) for the case with a discrete number of productivity types.

The borrower hires all workers in the economy and borrows as much as his borrowing constraint allows. The market clearing condition for debt is simply  $d_{Lt} + d_{Bt} = 0$ . Following similar steps as in the above model, we obtain our characterization of aggregate quantities.

**Proposition 6** *In the model with heterogenous discount factors, aggregate quantities satisfy*

$$\begin{aligned} Y_t &= AK_t^\alpha L^{1-\alpha} \\ K_{t+1} &= \bar{\beta}_t[\alpha Y_t + (1 - \delta)K_t] \\ \frac{C_{t+1}^E}{C_t^E} &= \frac{1 - \bar{\beta}_{t+1}}{1 - \bar{\beta}_t} \bar{\beta}_t \left[ \alpha \frac{Y_{t+1}}{K_{t+1}} + 1 - \delta \right] \\ \bar{\beta}_t &= \theta_{t-1}\beta_L + (1 - \theta_{t-1})\beta_B \end{aligned}$$

*Since  $\beta_L > \beta_B$ , a credit crunch modeled as a fall in  $\theta_{t-1}$ , is isomorphic to a fall in discount factors.*

The interpretation of this result is straightforward. Because borrowers and lenders have different discount factors, they accumulate wealth at different rates. The tighter the borrowing constraint, the larger the fraction of aggregate wealth that needs to be held by the borrower.<sup>22</sup> A credit crunch therefore has the same effect as a sudden decrease in the patience of the individuals in the economy.

## 5 Conclusion

The main finding of this paper is that one cannot hope to learn about the role of financial frictions in business cycle fluctuations using a representative agent framework and aggregate data alone. This follows from our result that the mapping from a credit crunch in a heterogenous agent economy to the aggregate variables in a representative agent economy depends crucially on the form of underlying heterogeneity; depending on where an economy features heterogeneity, a credit crunch can show up in very different aggregate variables. To make this argument concrete, we have examined the implications of a credit crunch for simple aggregate wedges. We have shown that a credit crunch shows up as an efficiency wedge if there is heterogeneity in the productivity of final goods producers. In contrast, it shows up as an investment wedge if investment costs are heterogenous; or as a labor wedge if recruitment costs are heterogenous.

Intuitively, a credit crunch worsens the allocation of resources across heterogenous individuals and this misallocation decreases the average efficiency of the technology in which entrepreneurs are heterogeneous. Importantly, in our framework a credit crunch does not affect aggregate investment in the sense that the Euler equation of a representative entrepreneur is undistorted. Hence it does not result in an (entrepreneurial) investment wedge unless investment costs are heterogenous implying a decrease in the average efficiency of investment goods production.

Another way of stating our main finding is that no given aggregate shock or wedge is a necessary condition for the importance of financial frictions in business cycle fluctuations. That is, the presence of financial frictions does not imply the presence of any given shock or wedge. It should be clear that, conversely, no given shock or wedge is a sufficient condition for the importance of financial frictions either. This is because there are many other possible drivers of aggregate shocks or wedges so none of them identify an economy that has been hit by a credit crunch.

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<sup>22</sup>For instance, in the extreme case with no credit markets,  $\theta_t = 0$ , the borrower is completely self-financed and he owns the entire capital stock of the economy.

# Appendices

## A Proofs

### A.1 Proof of Lemma 1

The Lemma follows directly from using the definitions of cash-on-hand,  $m_t$  and net worth,  $a_t$  in the dynamic programming problem 16.

### A.2 Proof of Lemma 2

The Lemma follows from the linearity of the portfolio allocation problem defining the function  $\tilde{m}_{t+1}(a', z)$ .

### A.3 Proof of Lemma 3

Consider the Bellman equation in (1). The Bellman equation can further be written as

$$V_t(m, z) = \max_{a'} \log(m - a') + \beta \mathbb{E}V_{t+1}(m_{t+1}(a', z), z')$$

$$m_{t+1}(a', z) = \tilde{m}_{t+1}(z)a', \quad \tilde{m}_{t+1}(z) = \max\{z\pi_{t+1} - r_{t+1} - \delta, 0\}\lambda_t + 1 + r_{t+1}$$

The proof proceeds with a guess and verify strategy. Guess that the value function takes the form  $V_t(m, z) = v_t(z) + B \log m$ , and substitute into the Bellman equation. In particular, note that  $\mathbb{E}V_t(m', z') = \mathbb{E}v_t(z') + B \log m'$ . The first order equation is

$$\frac{1}{m - a'} = \beta \frac{B}{\tilde{m}_{t+1}(z)a'} \tilde{m}_{t+1}(z) \quad \Rightarrow \quad a' = \frac{\beta B}{1 + \beta B} m$$

The Bellman equation becomes

$$v_t(z) + B \log m = \log \left[ \frac{1}{1 + \beta B} m \right] + \beta \left[ \mathbb{E}v_{t+1}(z') + B \log \frac{\beta B}{1 + \beta B} m \right]$$

Collecting the terms involving  $\log m$ , we can see that  $B = 1/(1 - \beta)$  and  $a' = \beta m$  as claimed.  $\square$

### A.4 Proof of Proposition 1

Consider first the bond market clearing condition. Using (18) and (19), we have that individual debt is  $d_{it+1} = (\lambda_t - 1)\beta m_{it}$  if  $z_{it+1} \geq \underline{z}_{t+1}$  and  $d_{it+1} = -\beta m_{it}$  otherwise. Using that  $z_{it+1}$  is independent of  $m_{it}$ , (9) becomes

$$(\lambda_t - 1) \int_{\underline{z}_{t+1}}^{\infty} \psi(z) dz - \int_0^{\underline{z}_{t+1}} \psi(z) dz = 0 \quad \text{or} \quad \lambda_t(1 - \Psi(\underline{z}_{t+1})) = 1. \quad (51)$$

Labor demand is

$$l_{it} = \left( \frac{\pi_t}{\alpha} \right)^{1/(1-\alpha)} k_{it} z_{it} \quad (52)$$

It follows that output is  $y_{it} = (\pi_t/\alpha) z_{it} k_{it}$ . Aggregate output is then

$$Y_t = \int y_{it} di = \frac{\pi_t}{\alpha} \int z_{it} k_{it} di.$$

Since  $k_{it} = \lambda_{t-1}a_{it} = \lambda_{t-1}\beta m_{it-1}$  if  $z_{it} \geq \underline{z}_t$  and zero otherwise, we have

$$\int z_{it}k_{it}di = \lambda_{t-1}X_t\beta M_{t-1} = \lambda_{t-1}X_tK_t, \quad X_t \equiv \int_{\underline{z}_t}^{\infty} z\psi(z)dz \quad (53)$$

Hence  $Y_t = (\pi_t/\alpha)\lambda_{t-1}X_tK_t$ . Next, consider the labor market clearing condition. Integrating (52) over all  $i$ ,

$$L = \left(\frac{\pi_t}{\alpha}\right)^{1/(1-\alpha)} \lambda_{t-1}X_tK_t. \quad (54)$$

Rearranging  $\pi_t = \alpha(\lambda_{t-1}X_t)^{\alpha-1}K_t^{\alpha-1}L^{1-\alpha}$  and using it the expression for output  $Y_t = (\lambda X_t)^\alpha K_t^\alpha L^{1-\alpha}$ . Eliminating  $\lambda_{t-1}$  using (51), we obtain (21). The law of motion for aggregate capital is derived by integrating (20) over all entrepreneurs:

$$K_{t+1} = \beta \left[ \pi_t \int z_{it}k_{it}di + (1-\delta)K_t \right] \quad (55)$$

Using (53) and (54),

$$K_{t+1} = \beta [\alpha Z_t K_t^\alpha L^{1-\alpha} + (1-\delta)K_t], \quad Z_t = (\lambda_t X_t)^\alpha,$$

which is equation (22) in Proposition 1.  $\square$

## A.5 Proof of Proposition 2

**Part (i):** That  $\tau_{X_t}^E = 0$  follows directly from inspection of (15) and (26).

**Part (ii):** Aggregate consumption is  $C_t = C_t^W + C_t^E$ . Hence aggregate consumption growth is

$$\frac{C_{t+1}}{C_t} = \frac{C_{t+1}^E}{C_t^E} \frac{C_t^E}{C_t} + \frac{C_{t+1}^W}{C_t^W} \frac{C_t^W}{C_t} = \frac{C_{t+1}^E}{C_t^E} + \frac{C_t^W}{C_t} \left( \frac{C_{t+1}^W}{C_t^W} - \frac{C_{t+1}^E}{C_t^E} \right)$$

Using (26),

$$\frac{C_{t+1}}{C_t} = \beta \left[ \alpha \frac{Y_{t+1}}{K_{t+1}} + 1 - \delta \right] + \frac{C_t^W}{C_t} \left( \frac{C_{t+1}^W}{C_t^W} - \frac{C_{t+1}^E}{C_t^E} \right)$$

Subtracting the equation defining the investment wedge (14) from both sides and rearranging, we obtain (27).  $\square$

## A.6 Proof of Lemma 5 (LHS)

We show in Appendix B that the saving policy function takes the form  $a_{it+1} = s_{t+1}(z_{it+1})m_{it}$  or  $k_{it+1} - d_{it+1} = s_{t+1}(z_{it+1})m_{it}$ . Aggregating over all types:

$$K_{t+1} = \bar{s}_{t+1}M_t, \quad \bar{s}_{t+1} \equiv \int_0^\infty s_{t+1}(z)\psi(z)dz$$

Since  $R_{it+1}^a = m_{it+1}/a_{it+1}$ , the individual Euler equations (28) can be written as

$$u'(c_{it}) = \beta \mathbb{E}[u'(c_{it+1})] \frac{m_{it+1}}{a_{it+1}}$$

Therefore

$$\int \frac{u'(c_{it})}{\beta \mathbb{E}[u'(c_{it+1})]} \frac{a_{it+1}}{K_{t+1}} di = \frac{M_{t+1}}{K_{t+1}}$$

We have that  $C_t^E = (1 - \bar{s}_{t+1})M_t$  and  $K_{t+1} = \bar{s}_{t+1}M_t$  and hence

$$\frac{C_{t+1}^E}{C_t^E} = \frac{1 - \bar{s}_{t+2}}{1 - \bar{s}_{t+1}} \frac{M_{t+1}}{M_t} = \frac{1 - \bar{s}_{t+2}}{1 - \bar{s}_{t+1}} \bar{s}_{t+1} \frac{M_{t+1}}{K_{t+1}}$$

Hence we obtain the desired result:

$$\int \frac{u'(c_{it})}{\beta \mathbb{E}[u'(c_{it+1})]} \frac{a_{it+1}}{K_{t+1}} di = \frac{M_{t+1}}{K_{t+1}} = \frac{C_{t+1}^E}{C_t^E} \frac{1}{\bar{s}_{t+1}} \frac{1 - \bar{s}_{t+1}}{1 - \bar{s}_{t+2}}. \square$$

### A.7 Proof of Proposition 3

**Part (i)** Combining Lemmas (4) and (5), the Euler equation for the representative entrepreneur is

$$\frac{C_{t+1}^E}{C_t^E} \frac{1}{\bar{s}_{t+1}} \frac{1 - \bar{s}_{t+1}}{1 - \bar{s}_{t+2}} = \alpha \frac{Y_{t+1}}{K_{t+1}} + 1 - \delta$$

The definition of the entrepreneurial investment wedge (14) can be written as

$$\frac{1}{\beta} \left( \frac{C_{t+1}^E}{C_t^E} \right)^\sigma (1 + \tau_{Xt}^E) = \alpha \frac{Y_{t+1}}{K_{t+1}} + (1 - \delta)(1 + \tau_{Xt+1}^E)$$

Combining the two equations implies (39).

**Part (ii)** Consider the definitions of the investment wedge and entrepreneurial investment wedge in (14) and (15). Using the CRRA functional form and subtracting one from the other, we get

$$\left( \frac{C_{t+1}}{C_t} \right)^\sigma (1 + \tau_{Xt}) - \left( \frac{C_{t+1}^E}{C_t^E} \right)^\sigma (1 + \tau_{Xt}^E) = \beta(1 - \delta)(\tau_{Xt+1} - \tau_{Xt+1}^E) \quad (56)$$

We have

$$\frac{C_{t+1}}{C_t} = \frac{C_{t+1}^E}{C_t^E} \frac{C_t^E}{C_t} + \frac{C_{t+1}^W}{C_t^W} \frac{C_t^W}{C_t} = \frac{C_{t+1}^E}{C_t^E} + \frac{C_t^W}{C_t} \left( \frac{C_{t+1}^W}{C_t^W} - \frac{C_{t+1}^E}{C_t^E} \right)$$

Substituting into (56), we obtain (40).  $\square$

### A.8 Proof of Proposition 4

Denote the Lagrange multiplier on the borrowing constraint (5) by  $\mu_{it}$  and the Lagrange multiplier on the constraint  $k_{it+1} \geq 0$  by  $\psi_{it}$ . The two Euler equations with respect to capital and debt are

$$\frac{1}{c_{it}} \omega_{it} = \beta \mathbb{E} \left[ \frac{1}{c_{it+1}} \right] [R_{t+1} + (1 - \delta)\omega_{it+1}] + \mu_{it}\theta_t + \psi_{it} \quad (57)$$

$$\frac{1}{c_{it}} = \beta \mathbb{E} \left[ \frac{1}{c_{it+1}} \right] (1 + r_{t+1}) + \mu_{it} \quad (58)$$

Multiply (57) by  $k_{it+1}$  and (58) by  $-d_{it+1}$  and add them

$$\frac{1}{c_{it}} [\omega_{it} k_{it+1} - d_{it+1}] = \beta \mathbb{E} \left[ \frac{1}{c_{it+1}} \right] [R_{t+1} k_{it+1} + (1-\delta) \omega_{it+1} k_{it+1} - (1+r_{t+1}) d_{it+1}] + \mu_{it} [\theta k_{it+1} - d_{it+1}] + \psi_{it} k_{it+1}$$

The complementary slackness condition corresponding to (5) is  $\mu_{it} [\theta k_{it+1} - d_{it+1}] = 0$  and  $\psi_{it} k_{it+1} = 0$ . It can then be verified that this Euler equation is satisfied by

$$k_{it+1} \omega_{it} - d_{it+1} = \beta m_{it}, \quad c_{it} = (1 - \beta) m_{it}$$

where  $m_{it} \equiv R_t k_{it} + (1 - \delta) \omega_{it} k_{it} - (1 + r_t) d_{it}$ .

$$C_t = (1 - \beta) \left[ R_t K_t + (1 - \delta) \int \omega_{it} k_{it} di \right] \quad (59)$$

$$\int \omega_{it} k_{it+1} di = \beta \left[ R_t K_t + (1 - \delta) \int \omega_{it} k_{it} di \right]$$

$$K_{t+1} = \beta \left[ \int \omega_{it} \frac{k_{it+1}}{K_{t+1}} di \right]^{-1} \left[ R_t K_t + (1 - \delta) \int \omega_{it} k_{it} di \right] \quad (60)$$

Combining (59) and (60) yields

$$\frac{C_{t+1}}{C_t} \int \omega_{it} \frac{k_{it+1}}{K_{t+1}} di = \beta \left[ R_{t+1} + (1 - \delta) \int \omega_{it+1} \frac{k_{it+1}}{K_{t+1}} di \right]$$

Using that  $R_{t+1} = \alpha Y_{t+1} / K_{t+1}$ , this is (41).

## A.9 Proof of Lemma 6

The steps described here follow Shimer (2010). We modify his derivations to allow for heterogeneity on the side of employers. Let  $V_l(l_{it}, d_{it}, \omega_{it}, t)$  denote the marginal utility for entrepreneur  $i$  with employment  $l_{it}$ , debt  $d_{it}$ , and recruitment cost,  $\omega_{it}$  of employing a worker at wage  $w_{it}$ . Let  $W_i(\{l_{it}\}, t)$  denote the marginal utility for workers at the equilibrium level of employment of having one worker employed at a wage  $w_{it}$  in period  $t$  rather than unemployed.<sup>23</sup> We are interested in characterizing wages that that a worker is willing to accept,  $\geq 0$  and that all entrepreneurs are willing to pay,  $V_l(l_{it}, d_{it}, \omega_{it}, t) \geq 0$  for all  $i$ .

Consider first the value of an entrepreneur which is given by

$$\begin{aligned} V_l(l, d, \omega) &= \max_{c, x, d'} \log c + \beta \mathbb{E} V(l', d', \omega') \quad \text{s.t.} \\ c + \omega x - d' &= Al - w_t l - (1 + r_t) d, \\ l' &= (1 - \delta) l + x, \quad x \geq 0, \quad d' \leq \phi A l' \end{aligned}$$

The first order condition for recruiting,  $x_{it}$ , is

$$\omega_{it} \frac{1}{c_{it}} = \beta \mathbb{E} V_l(l_{it+1}, d_{it+1}, \omega_{it+1}, t + 1) \quad (61)$$

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<sup>23</sup>As shown below, this value depends on the entire distribution of employment,  $\{l_{it}\}$

and the envelope condition

$$V_l(l_{it}, d_{it}, \omega_{it}, t) = \frac{A_t + (1 - \delta)\omega_{it} - w_{it}}{c_{it}} \quad (62)$$

This is the marginal value to an entrepreneur of having an extra worker paid  $w_{it}$

Next, consider workers. Workers take as given the distribution of employment and its evolution of employment. In particular, they take as given the (exogenous) job separation rate  $\delta$  and the (endogenous) probability of finding a job at firm  $i$ ,  $f_{it}$ . This job finding rate is defined by the requirement that the number of workers finding jobs,  $f_{it}(1 - L_t)$ , is equal to the number of workers recruited by firms  $x_{it}$  and hence  $f_{it} = x_{it}/(1 - L_t)$ . From the point of view of workers employment then evolves as  $l_{it+1} = (1 - \delta)l_{it} + f_{it}(1 - L_t)$ . The value of a worker can then be written in recursive form as

$$W(\{l_{it}\}, t) = \log \left( \int w_{it} l_{it} di \right) - \gamma \int l_{it} di + \beta \mathbb{E}W(\{l_{it+1}\}, t + 1)$$

The envelope condition is

$$W_i(\{l_{it}\}, t) = \frac{w_{it}}{C_t^W} - \gamma + \beta(1 - \delta)\mathbb{E}W_i(\{l_{it+1}\}, t + 1) - \beta \int f_{jt}\mathbb{E}W_j(\{l_{it+1}\}, t + 1) dj \quad (63)$$

### A.9.1 Common Wage

We now show that a *common* wage satisfying the condition in Lemma (6) is always in the bargaining set of all entrepreneurs and workers. From (62), it is easy to see that  $w_t \leq A$  implies  $V_l(l_{it}, d_{it}, \omega_{it}, t) \geq 0$ . Further, note that with a *common* wage, the distribution of employment no longer matters for the household and hence  $W_i(\{l_{it}\}, t) = W_L(L_t, t)$  for all  $i$  where.

$$W_L(L_t, t) = \frac{w_t}{C_t^W} - \gamma + \beta \left( 1 - \delta - \int f_{jt} dj \right) W_L(L_{t+1}, t + 1)$$

Then  $w_t \geq \gamma C_t^W$  for all  $t$  implies that  $W_L(L_t, t) \geq 0$  for all  $t$ . This proves Lemma 6.  $\square$

### A.9.2 Generalized Nash Bargaining: Entrepreneur-Specific Wage

Following the same analysis as in Shimer (2010), it can easily be shown that if wages are determined by generalized Nash bargaining, the entrepreneur-specific wage  $w_{it}$  satisfies

$$(1 - \phi)W_i(\{l_{it}\}, t)C_t^W = \phi V_l(l_{it}, d_{it}, \omega_{it}, t)c_{it} \quad (64)$$

where  $\phi \in [0, 1]$  represents the worker's bargaining power. Multiply (63) by  $(1 - \phi)C_t^W$  to obtain

$$(1 - \phi)W_i(\{l_{it}\}, t)C_t^W = (1 - \phi)(w_{it} - \gamma C_t^W) + \frac{C_t^W}{C_{t+1}^W} \left[ \beta(1 - \delta)C_{t+1}^W(1 - \phi)\mathbb{E}W_i(\{l_{it+1}\}, t + 1) - \beta \int f_{jt}C_{t+1}^W(1 - \phi)\mathbb{E}W_j(\{l_{it+1}\}, t + 1) dj \right]$$

Substitute in from (64)

$$\phi V_l(l_{it}, d_{it}, \omega_{it}, t)c_{it} = (1 - \phi)(w_{it} - \gamma C_t^W) + \frac{C_t^W}{C_{t+1}^W} \left[ \beta(1 - \delta)\phi\mathbb{E}V_l(l_{it+1}, d_{it+1}, \omega_{it+1}, t + 1)c_{it+1} - \beta \int f_{jt}\phi\mathbb{E}V_l(l_{jt+1}, d_{jt+1}, \omega_{jt+1}, t + 1)c_{jt+1} dj \right]$$

Using that from (61)

$$\mathbb{E}V_l(l_{it+1}, d_{it+1}, \omega_{it+1}, t+1)c_{it+1} = \frac{c_{it+1}}{c_{it}}\omega_{it}$$

to eliminate  $V_l(l_{it+1}, d_{it+1}, \omega_{it+1}, t+1)$  and (62) to eliminate  $V_l(l_{it}, d_{it}, \omega_{it}, t)$ ,

$$\phi[A + (1 - \delta)\omega_{it} - w_{it}] = (1 - \phi)(w_{it} - \gamma C_t^W) + \frac{C_t^W}{C_{t+1}^W}\phi \left[ (1 - \delta)\frac{c_{it+1}}{c_{it}}\omega_{it} - \int \frac{c_{jt+1}}{c_{jt}}f_{jt}\omega_{jt}dj \right]$$

Rearranging

$$\phi \left[ A + (1 - \delta)\omega_{it} \left( 1 - \frac{C_t^W}{C_{t+1}^W} \frac{c_{it+1}}{c_{it}} \right) + \frac{C_t^W}{C_{t+1}^W} \int \frac{c_{jt+1}}{c_{jt}}f_{jt}\omega_{jt}dj - w_{it} \right] = (1 - \phi)(w_{it} - \gamma C_t^W)$$

And hence

$$w_{it} = \phi \left( A + (1 - \delta)\omega_{it} \left( 1 - \frac{C_t^W}{C_{t+1}^W} \frac{c_{it+1}}{c_{it}} \right) + \frac{C_t^W}{C_{t+1}^W} \int \frac{c_{jt+1}}{c_{jt}}f_{jt}\omega_{jt}dj \right) + (1 - \phi)\gamma C_t^W$$

This is the Nash-bargaining solution. Note that wages are entrepreneur-specific because of heterogeneity in recruitment costs, and also because financing constraints imply that consumption growth rates differ across entrepreneurs.<sup>24</sup>

## A.10 Proof of Lemma 7

Defining “cash-on-hand”

$$m_{it} \equiv Al_{it} + (1 - \delta)\omega_{it}l_{it} - w_{it}l_{it} - (1 + r_t)d_{it},$$

the budget constraint of an entrepreneur becomes  $c_{it} - d_{it+1} + \omega_{it}l_{it+1} = m_{it}$ . The problem of an entrepreneur can then be stated in recursive form as

$$V(m, \omega) = \max_{l', d'} \log(m - \omega l' + d') + \beta \mathbb{E}V(m', \omega') \quad \text{s.t.}$$

$$m' = Al' + (1 - \delta)\omega' l' - \omega l' - (1 + r)d', \quad d' \leq \phi Al'$$

Following similar steps as in the proof of Lemma 3, we can show that entrepreneurs save a constant fraction  $\beta$  of their cash-on-hand,  $m_{it}$ , and hence that his optimal labor choice satisfies (49). $\square$

## A.11 Proof of Proposition 5

The proposition follows directly from aggregating (49) across all entrepreneurs.

## A.12 Proof of Proposition 6

Following similar steps as in the proof of proposition (3), one can show that with log-utility individuals save a constant fraction of their cash-on-hand

$$a_{Bt+1} = \beta_B \left[ (A^{1/\alpha} \pi_t + 1 - \delta)K_t - (1 + r_t)d_{Bt} \right], \quad a_{Lt+1} = \beta_L(1 + r_t)a_{Lt}$$

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<sup>24</sup>Without heterogeneity and with perfect financial markets (implying  $c_{it+1}/c_{it} = C_{t+1}^W/C_t^W$ ), the wage would simply be  $w_t = \phi(A + f\omega) + (1 - \phi)\gamma C_t^W$ .

where the net worth of borrowers and lenders is  $a_{Bt+1} = K_{t+1} - d_{Bt+1}$  and  $a_{Lt+1} = -d_{Lt+1}$ , and  $\pi_t = \alpha((1 - \alpha)/w_t)^{1/(1-\alpha)}$  as above. Because the environment is deterministic,  $r_t \rightarrow 1/\beta_L - 1$  as  $t \rightarrow \infty$ . Suppose the borrowers' borrowing constraint were not binding. Then  $A^{1/\alpha}\pi_t = r_t + \delta$  and hence  $a_{Bt+1} = \beta_B(1 + r_t)a_{Bt}$ . Since  $\beta_B < \beta_L$ ,  $\beta_B(1 + r_t) < 1$  and hence  $a_{Bt} \rightarrow 0$  as  $t \rightarrow \infty$ . But then the borrowing constraint must be binding which is a contradiction. Using that the borrowing constraint, (50), binds

$$(1 + r_t)d_{Bt} = -(1 + r_t)d_{Lt} = \theta_{t-1}[A^{1/\alpha}\pi_t + 1 - \delta]K_t$$

So

$$K_{t+1} - d_{Bt+1} = \beta_B(1 - \theta_{t-1})[A^{1/\alpha}\pi_t + 1 - \delta]K_t, \quad -d_{Lt+1} = \beta_L\theta_{t-1}[A^{1/\alpha}\pi_t + 1 - \delta]K_t$$

Summing

$$K_{t+1} = [(1 - \theta_{t-1})\beta_B + \theta_{t-1}\beta_L][A^{1/\alpha}\pi_t + 1 - \delta]K_t \quad (65)$$

Since only borrowers hire workers, we have  $\pi_t = \alpha(A^{1/\alpha}K_t)^{\alpha-1}L^{1-\alpha}$ . Substituting back into (65), we obtain the expressions in Proposition 6.  $\square$

## B Analysis of the general CRRA case

In this appendix we consider a more general process of entrepreneurial productivity that allows for the persistence of shocks over time. In particular, we assume that in each period entrepreneurs retain their productivity with probability  $\gamma$ . With the complementary probability  $1 - \gamma$  entrepreneurs draw a new productivity from the distribution  $\psi(z)$ .

### B.1 Characterization of Individual's Saving Problem

The value function of an entrepreneur with cash-in-hand  $m$  and ability  $z$  solve

$$V_t(m, z) = \max_{a'} \frac{(m - a')^{1-\sigma}}{1 - \sigma} + \beta \mathbb{E} [V_{t+1}(m_{t+1}(a', z), z') | z]$$

where  $m_{t+1}(a', z) = \tilde{m}_{t+1}(z)a'$ ,  $\tilde{m}_{t+1}(z) = \max\{z\pi_{t+1} - r_{t+1} - \delta, 0\}\lambda_t + 1 + r_{t+1}$ .

The proof proceeds with a guess and verify strategy. Guess that the value function takes the form  $V_t(m, z) = v_t(z)\frac{m^{1-\sigma}}{1-\sigma}$ , and substitute into the Bellman equation.

$$v_t(z)\frac{m^{1-\sigma}}{1-\sigma} = \max_{a'} \frac{(m - a')^{1-\sigma}}{1-\sigma} + \beta \mathbb{E} [v_{t+1}(z') | z] \frac{[\tilde{m}_{t+1}(z)a']^{1-\sigma}}{1-\sigma}$$

It will be useful to define the auxiliary variable

$$\nu_{t+1}(z) = \beta \mathbb{E} [v_{t+1}(z') | z] \tilde{m}_{t+1}(z)^{1-\sigma} \quad (66)$$

so that the Bellman equation is

$$v_t(z)\frac{m^{1-\sigma}}{1-\sigma} = \max_{a'} \frac{(m - a')^{1-\sigma}}{1-\sigma} + \nu_{t+1}(z)\frac{(a')^{1-\sigma}}{1-\sigma} \quad (67)$$

The first order condition is

$$(m - a')^{-\sigma} = \nu_{t+1}(z)(a')^{-\sigma}$$

or

$$a' = s_{t+1}(z)m, \quad s_{t+1}(z) \equiv \frac{1}{1 + \nu_{t+1}(z)^{-1/\sigma}}$$

Consumption is

$$c = \frac{\nu_{t+1}(z)^{-1/\sigma}}{1 + \nu_{t+1}(z)^{-1/\sigma}}m$$

Substituting into the Bellman equation (67) and canceling the terms involving  $m^{1-\sigma}/(1-\sigma)$ ,

$$v_t(z) = \left( \frac{\nu_{t+1}(z)^{-1/\sigma}}{1 + \nu_{t+1}(z)^{-1/\sigma}} \right)^{1-\sigma} + \nu_{t+1}(z) \left( \frac{1}{1 + \nu_{t+1}(z)^{-1/\sigma}} \right)^{1-\sigma}$$

which after some manipulation becomes

$$v_t(z) = \left( 1 + \nu_{t+1}(z)^{1/\sigma} \right)^\sigma$$

or using the definition of  $\nu_{t+1}(z)$  in (66),

$$v_t(z) = \left( 1 + \{ \beta \mathbb{E} [v_{t+1}(z')|z] \tilde{m}_{t+1}(z)^{1-\sigma} \}^{1/\sigma} \right)^\sigma$$

This is a functional equation in  $v_t(z)$  that can be solved numerically.

## B.2 Evolution of the Wealth Density, Aggregate Capital and Productivity

The evolution of the wealth density  $\xi_t(z)$  is described by the following functional equation

$$\xi_{t+1}(z) = \frac{K_t}{K_{t+1}} \left[ \gamma s_{t+1}(z) \tilde{m}_t(z) \xi_t(z) + (1-\gamma) \psi(z) s_{t+1}(z) \int \tilde{m}_t(z_{-1}) \xi(z_{-1}) dz_{-1} \right] \quad (68)$$

Using Lemma 4 and integrating over all  $z$  we obtain a law of motion for aggregate capital

$$K_{t+1} = \gamma K_t \int s_{t+1}(z) \tilde{m}_t(z) \xi_t(z) dz + (1-\gamma) \bar{s}_{t+1} [\alpha Y_t + (1-\delta) K_t]. \quad (69)$$

There are two cases for which the model allows for a simple aggregation, given the evolution of aggregate productivity  $Z_t$ . First, if we assume that entrepreneurs' productivity is iid over time, equation B.2 specializes to

$$K_{t+1} = \bar{s}_{t+1} [\alpha Y_t + (1-\delta) K_t].$$

The second correspond to the case of log preferences. Using that  $s_{t+1}(z) = \bar{s}_{t+1} = \beta$  and applying Lemma 4 to the first term in the right hand side of equation B.2 we obtain a simple equation describing the evolution of aggregate capital:

$$K_{t+1} = \beta [\alpha Y_t + (1-\delta) K_t].$$

While we can aggregate the model given the evolution of aggregate productivity, in the more general model the evolution of aggregate productivity is itself a function of the wealth density. In particular, aggregate productivity is a capital weighted average of entrepreneurs' productivity

$$Z_t = \left( \frac{\int_{\underline{z}_t}^{\infty} z \xi(z) dz}{1 - \Xi(\underline{z}_t)} \right)^\alpha$$

The cutoff is defined by

$$\lambda_{t-1}(1 - \Xi(\underline{z}_t)) = 1.$$

## C Model with Homogenous Recruitment Costs and Heterogenous Productivity

Consider the same model as in section 3 but where entrepreneurs are heterogenous in their productivity,  $y_{it} = z_{it}l_{it}$ ,  $z_{it}$  is drawn from  $\psi(z)$  iid over time and across entrepreneurs. Everything remains unchanged except the budget constraint of an entrepreneur which now is

$$c_{it} + x_{it} - d_{it+1} = z_{it}l_{it} - w_t l_{it} - (1 + r_t)d_{it}$$

The equilibrium has the feature that there is a productivity cutoff for being active  $\underline{z}_t$ . Only entrepreneurs who are above this cutoff are active. Hence the equivalent of the sufficient condition in Lemma 6 for a common wage,  $w_t$ , to be in the bargaining set is  $\gamma C_t^W \leq w_t \leq \underline{z}_t$ . We again impose that the wage lies halfway between these bounds:

$$w_t = \frac{\gamma C_t^W + \underline{z}_t}{2} \Rightarrow w_t = \frac{\underline{z}_t}{2 - \gamma L_t}$$

where the second equality follows because  $C_t^W = w_t L_t$ .

Defining cash-on-hand  $m_{it} = z_{it}l_{it} + (1 - \delta)l_{it} - w_t l_{it} - (1 + r)d_{it}$  and net worth  $a_{it+} = l_{it+1} - d_{it+1}$ , the Bellman equation of an entrepreneur is

$$V(m, z) = \max_{a', l', d'} \log(m - a') + \beta \mathbb{E}V(\tilde{m}(a', z), z')$$

$$\tilde{m}(a', z) = \max_{l', k'} z l' + (1 - \delta)l - w' l' - (1 + r')d', \quad l' - d' = a', \quad l' \leq \lambda(z)a', \quad \lambda(z) = \frac{1}{1 - \theta z}$$

Optimal labor choice therefore satisfies

$$l_{it+1} = \begin{cases} \lambda(z_{it+1})a_{it+1}, & z_{it+1} \geq \underline{z}_{t+1} \\ 0, & z_{it+1} < \underline{z}_{t+1} \end{cases} \quad (70)$$

where  $\underline{z}_{t+1} = w_{t+1} - 1 + \delta$ . We can again show that the assumption of log-utility implies that agents save a constant fraction of cash-on-hand,  $a_{it+1} = \beta m_{it}$  or

$$l_{it+1} - d_{it+1} = \beta [z_{it}l_{it} + (1 - \delta)l_{it} - w_t l_{it} - (1 + r)d_{it}] \quad (71)$$

Next we can find an expression for the productivity cutoff,  $\underline{z}_t$ . From (70), we have

$$L_t = \int l_{it} di = \int_{\underline{z}_t}^{\infty} \lambda(z)\psi(z)dz \beta M_{t-1} = \int_{\underline{z}_t}^{\infty} \lambda(z)\psi(z)dz L_t$$

Hence the cutoff,  $\underline{z}_t$ , is pinned down from  $\int_{\underline{z}_t}^{\infty} \lambda(z)\psi(z)dz = 1$ . Aggregating over all entrepreneurs and using (70) gives

$$L_{t+1} = \beta [Z_t + 1 - \delta - w_t] L_t, \quad w_t = \frac{\underline{z}_t}{2 - \gamma L_t} \quad \text{where} \quad Z_t = \int_{\underline{z}_t}^{\infty} z \lambda(z)\psi(z)dz$$

is TFP. Note that employment, and hence the labor wedge, only move because of movements in TFP.

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