# Deprivation and Social Exclusion*

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# Abstract

Social exclusion manifests itself in the persistent lack of an individual's access to functionings as compared to other members of society and we model it as being in a state of deprivation over time. We view deprivation as having two basic determinants: the lack of identification with other members of society and the aggregate alienation experienced by an agent with respect to those with fewer functioning failures. Using an axiomatic approach, we characterize new individual and aggregate measures of deprivation and of social exclusion. The aggregate measures are then applied to EU data for the period from 1994 to 2001. *Journal of Economic Literature* Classification No.: D63.



**Keywords:** Social Exclusion, Deprivation, Equity.

# 1 Introduction

This paper is concerned with the measurement of social exclusion and its relation with deprivation. The term social exclusion has its origins in Lenoir (1974; see also Lenoir, 1989). Lenoir, then Secrétaire d'Etat à l'Action Sociale in the Chirac government, referred to the excluded as consisting not only of the poor but of a wide variety of people, namely the social misfits. The meaning of the term evolved and expanded in the following years to include all individuals and groups that are wholly or partly prevented from participation in their society and in various aspects of cultural and community life.

Social exclusion is of increasing interest because it has gained a primary role in official documents and in the political debate in Europe and, more recently, in Australia, Canada and the United States. In the Treaty of Amsterdam, signed in 1997, the European Union included the reduction of social exclusion among its objectives. During the Lisbon European Council of March 2000, the Union formulated the goal "to become the most competitive dynamic knowledge-based economy in the world capable of sustainable economic growth with more and better jobs and greater social cohesion" (European Council, 2000). Clearly, achieving this goal involves the design of policies aimed at combating social exclusion. Both providing a rigorous conceptual foundation and giving some guidance as to the application of the concepts suggested here are of importance because of the public-policy relevance of the issue.

We are not the first to propose a measure of social exclusion. Several attempts have already been made by scholars in various subdisciplines of the social sciences; see, for example, Burchardt, Le Grand and Piachaud (1999), Bradshaw, Williams, Levitas, Pantazis, Patsios, Townsend, Gordon and Middleton (2000), Whelan, Layte and Maître (2001), Tsakloglou and Papadopoulos (2002), Chakravarty and D'Ambrosio (2003) and Poggi (2003). As is the case for Chakravarty and D'Ambrosio (2003), we employ the axiomatic method and, to the best of our knowledge, theirs is the only other paper based on it. While their approach and ours exhibit some similarities in the basic setup we use, there are substantial differences in how aspects of social exclusion are taken into consideration. Consequently, the measures we characterize here are very different from those proposed in their contribution.

Social exclusion has received a considerable amount of attention among social scientists discussing the attributes, differences and novelties of it with respect to more traditional concepts such as income poverty, multidimensional poverty and inequality. See, for example, Duffy (1995), Room (1995), Atkinson (1998), Klasen (1998), Rowntree Foundation

(1998), Mejer (2000), Sen (2000), Atkinson, Cantillon, Marlier and Nolan (2002), just to mention a few. This paper has a different focus with respect to these earlier contributions: we do not discuss in depth the vagueness of the concept and its characteristics, but our theoretical contribution takes for granted the elements characterizing social exclusion that resulted from these debates, and build on those proposing a measure of the phenomenon. At the same time our empirical contribution in calculating the measure takes for granted the variables that Eurostat (2000) deemed appropriate to measure social exclusion in EU member states.

The most fundamental elements identifying the notion of social exclusion are multi-dimensional functioning failure, relativity and dynamic considerations.

Social exclusion is a multi-dimensional concept that covers economic, social and political aspects: it deals with the failure to attain adequate levels of various functionings (Sen, 1985) that are deemed valuable in the society under analysis.

Social exclusion is a relative concept in the sense that an individual can be socially excluded only in comparison with other members of a society: there is no 'absolute' social exclusion, and an individual can be declared socially excluded only with respect to the society it is considered to be a member of. An additional relative feature is that social exclusion depends on the extent to which an individual is able to associate and identify with others.

The relativity element of social exclusion makes the latter closely related to the concept of deprivation. Runciman (1966) formulates the idea that a person's feeling of deprivation in a society arises out of comparing its situation with those who are better off: "The magnitude of a relative deprivation is the extent of the difference between the desired situation and that of the person desiring it" (Runciman, 1966, p.10). This intuition was used by Sen (1976), Yitzhaki (1979) and others to obtain measures of deprivation with income as the relevant variable. In this paper we extend the framework established in the earlier literature.

While the concept of deprivation is usually treated as a static concept, social exclusion has important dynamic aspects: an individual can become socially excluded if its condition of deprivation is persistent or worsens over time. Therefore, the measurement of social exclusion requires the inclusion of time as an important variable (see also Atkinson, 1998). Indeed, our distinction between deprivation and social exclusion is captured by this temporal aspect. An individual experiences a higher degree of social exclusion in situations where deprivation is present in consecutive periods as compared to equal levels of deprivation interrupted by periods without deprivation. It is important to note a difference between deprivation measured in terms of incomes and our approach where functioning failures determine the degree of deprivation. If income is the variable relevant for deprivation, most individuals have a positive degree of deprivation in every period because only those in the top-income group do not suffer from deprivation at all. In that case, there is not much point in paying attention to situations where deprivation is equal to zero because this phenomenon occurs so infrequently that it can be neglected. In contrast, if deprivation is measured in terms of functioning failures, a substantial proportion of the population is not deprived in some periods because deprivation with respect to access to basic functionings is considered.

The axiomatic part of our study focuses on the construction of new individual and aggregate measures of deprivation and of social exclusion. First, we characterize a new class of individual measures of deprivation based on functioning-failure profiles as the primary data. The difference between our measures and those appearing in the earlier literature is that, in addition to the aggregate alienation experienced by the agent with respect to those who have fewer functioning failures, our indices also depend on the capacity of an individual to identify with other members of society. Following our characterization of individual measures of deprivation, we axiomatize an aggregate index of deprivation that measures deprivation in a society as a whole. Next, we use profiles of functioning failures over time to obtain individual measures of social exclusion. We take into consideration the temporal pattern of the individual levels of deprivation and assign higher values to an individual index of social exclusion if the condition of deprivation persists over time. To conclude the theoretical part of our study, an aggregate measure of social exclusion is characterized. Our characterizations of individual and aggregate measures of deprivation are of interest in their own right, in addition to providing the basis for our study of social exclusion. There seem to have been rather few axiomatizations of deprivation indices in the literature, and this paper provides a contribution towards filling that gap.

In the empirical part of our study, we calculate aggregate deprivation and social exclusion using EU data from 1994 to 2001. We carry out a comparative study among several countries with respect to the degree of deprivation and social exclusion they experienced in that period.

The remainder of the paper is organized as follows. We begin in Section 2 with a discussion of our formal framework and the notion of functioning failures as the basic ingredients of our analysis. Section 3 contains an axiomatization of a class of individual measures of deprivation in terms of functioning failures. Moreover, we characterize aggregate deprivation as the arithmetic mean of the individual levels of deprivation. In Section

4, a characterization of a class of individual measures of social exclusion is presented and, in addition, our axiomatic analysis is concluded by characterizing an aggregate measure of social exclusion as the arithmetic mean of the individual measures. In Section 5, we describe the empirical results for data in EU member states covering the period 1994 to 2001. Section 6 concludes.

## 2 Functioning-failure profiles

We use $\mathbb{N}$ to denote the set of all positive integers and $\mathbb{R}$ ($\mathbb{R}_+$, $\mathbb{R}_{++}$) is the set of all (all non-negative, all positive) real numbers. For a non-empty and finite set $M \subseteq \mathbb{N}$, the set $\mathbb{R}^M_+$ is the set of $|M|$-dimensional vectors of non-negative real numbers whose components are labelled by the elements in $M$. Furthermore, we define $\mathcal{N} = \mathbb{N} \setminus \{1\}$. $\mathcal{P}$ is the set of all finite subsets of $\mathbb{N}$ with at least two elements. For $n \in \mathbb{N}$, $\mathbf{1}_n$ is the vector consisting of $n$ ones. Agents are indexed by positive integers, and $N \in \mathcal{P}$ is the set of individuals in a society.

We assume that, for each individual, there exists a measure of functioning failure which indicates the degree to which functionings that are considered relevant are not available to the agent. The individual functioning failures constitute the primary inputs for our analysis. As a consequence of this modelling choice, the multi-dimensional aspect of social exclusion is not explicitly taken into consideration because we assume that a first aggregation step has already been performed in order to arrive at this single measure of functioning failure. On the other hand, this means that our approach is very flexible because it is compatible with any way in which this aggregation may be performed. A plausible possibility for such a measure is the number of functioning failures, which is the measure used in our empirical application. However, we use a more general approach that assumes the set of possible values of a measure of functioning failure to be non-negative real numbers. This is plausible because functioning failures could be partial, or the measures could incorporate weights that reflect the relative importance of these functioning failures. Our characterization results remain true if, instead of all non-negative real numbers, only rational numbers are allowed as functioning-failure values but, for simplicity, we work with the more standard domain of all real numbers.

For an individual $i \in \mathbb{N}$, $q_i \in \mathbb{R}_+$ is the functioning failure suffered by $i$ in a given period. To avoid unnecessary notational complexities at this stage, the period under consideration is not identified explicitly until later on when we move from deprivation to social exclusion. Let $\Omega = \cup_{N \in \mathcal{P}} \mathbb{R}^N_+$. A functioning-failure profile is a vector $q \in \Omega$. Let

$q, \bar{q} \in \cup_{N \in \mathcal{P}} \mathbb{R}^N_+$ and suppose $M \subset N$ is non-empty. The vector $q_M \in \mathbb{R}^M_+$ is defined by $q_M = (q_i)_{i \in M}$ and, analogously, $q_{-M} \in \mathbb{R}^{N \setminus M}_+$ is $q_{-M} = (q_i)_{i \in N \setminus M}$. $(q_{-M}, \bar{q}_M) \in \mathbb{R}^N_+$ is given by $(q_{-M}, \bar{q}_M)_i = q_i$ if $i \in N \setminus M$ and $(q_{-M}, \bar{q}_M)_i = \bar{q}_i$ if $i \in M$. For $i \in \mathbb{N}$, let $\mathcal{P}_i \subseteq \mathcal{P}$ be the set of all $N \in \mathcal{P}$ with $i \in N$, and let $\Omega_i = \cup_{N \in \mathcal{P}_i} \mathbb{R}^N_+$.

Our axiomatization proceeds as follows. First, we characterize a class of individual measures of deprivation the domain of which is the set of possible functioning-failure profiles. In addition, we characterize an aggregate measure of deprivation. Next, we move from individual measures of deprivation to individual measures of social exclusion. An important aspect that distinguishes social exclusion from deprivation is the dynamic aspect of social exclusion. Consequently, our notion of social exclusion is based on the development of functioning-failure profiles over time, and we characterize a class of individual measures of social exclusion with a set of plausible axioms. In a final step, we axiomatize an aggregate measure of social exclusion based on these individual indicators.

# 3 Individual and aggregate measures of deprivation

An individual deprivation index for individual $i \in N$ is a function $D_i \colon \Omega_i \to \mathbb{R}_+$. For $N \in \mathcal{P}_i$ and $q \in \mathbb{R}^N_+$, $D_i(q)$ is the degree of deprivation suffered by individual $i$ in profile $q$. The set of individuals whose functioning failure is lower than that of $i$ in $q$ is $\mathcal{B}_i(q) = \{j \in N \mid q_j < q_i\}$.

We now formulate some desirable properties of $D_i$. The first of them is a normalization axiom. We use zero as the minimal value of the deprivation index and, due to the relative nature of the notion of deprivation, we assume that this minimal value of $D_i$ is obtained whenever no one in society has fewer functioning failures than individual $i$. That is, $D_i(q)$ is equal to zero whenever the set $\mathcal{B}_i(q)$ is empty. Conversely, we require the degree of individual $i$'s deprivation to be positive whenever there are people who experience fewer functioning failures than $i$. As a result, our normalization axiom requires that the deprivation of individual $i$ is zero if and only if the set of individuals with fewer functioning failures is empty—that is, if and only if $i$'s functioning failures are minimal within the profile under consideration.

**Normalization.** For all $q \in \Omega_i$,

$$D_i(q) = 0 \Leftrightarrow \mathcal{B}_i(q) = \emptyset.$$

In determining the degree of deprivation suffered by an individual $i$, it is natural to assume that $i$'s deprivation depends only on its own functioning failures and on those of the individuals who have fewer functioning failures than $i$, that is, those in $\mathcal{B}_i(q)$. The idea that a person's feeling of deprivation in a society arises out of comparing its situation with those who are better off has first been formulated by Runciman (1966) and then used by Yitzhaki (1979). Yitzhaki argues that individual $i$'s level of deprivation is an increasing function of the number of people who are better off than $i$. We adapt this general idea to our framework and assume that the extent to which an individual considers itself deprived does not depend on the situation of individuals who have a degree of functioning failure equal to or exceeding that of the individual itself. The resulting axiom is analogous to the focus axiom used in poverty measurement (see Sen, 1976).

**Focus.** For all $N \in \mathcal{P}_i$ and for all $q, \bar{q} \in \mathbb{R}^N_+$, if $\mathcal{B}_i(\bar{q}) = \mathcal{B}_i(q)$ and $\bar{q}_j = q_j$ for all $j \in \mathcal{B}_i(q) \cup \{i\}$, then

$$D_i(\bar{q}) = D_i(q).$$

As usual, anonymity requires that the identities of the individuals are irrelevant in obtaining a social index. For the individual index $D_i$, however, it is clear that individual $i$ itself may (and usually will) play a special role. Thus, the anonymity axiom we employ is restricted to the set of individuals other than $i$ and we obtain the following conditional version.

**Conditional anonymity.** For all $N, M \in \mathcal{P}_i$ such that $|N| = |M|$, for all bijections $\rho\colon M \to N$ such that $\rho(i) = i$, for all $q \in \mathbb{R}^N_+$ and for all $\bar{q} \in \mathbb{R}^M_+$, if $\bar{q}_j = q_{\rho(j)}$ for all $j \in M$, then

$$D_i(\bar{q}) = D_i(q).$$

The next axiom is a standard condition in much of economic theory. It is (linear) homogeneity, which requires that if a profile is multiplied by a positive number, then the corresponding level of deprivation is multiplied by the same number. This property ensures that a proportional change in the profile of functioning failures leads to an equiproportional change in deprivation.

**Homogeneity.** For all $q \in \Omega_i$ and for all $\lambda \in \mathbb{R}_{++}$,

$$D_i(\lambda q) = \lambda D_i(q).$$

Translation invariance imposes a restriction on the response of an index to equal absolute changes in a profile. If the same number is added to each functioning failure, the value of the deprivation index is unchanged. We employ a stronger axiom that applies to additions of different numbers, provided that the set of individuals with fewer functioning failures than $i$ is unchanged and, moreover, the value added to the functioning failures of those that are equally well or worse off than $i$ is the arithmetic mean of the values added to those in $\mathcal{B}_i(e)$.

**Strong translation invariance.** For all $N \in \mathcal{P}_i$, for all $q, \bar{q} \in \mathbb{R}^N_+$ and for all $\delta \in \mathbb{R}^{\mathcal{B}_i(q)}$, if $\mathcal{B}_i(\bar{q}) = \mathcal{B}_i(q) \neq \emptyset$, $\bar{q}_j = q_j + \delta_j$ for all $j \in \mathcal{B}_i(q)$ and $\bar{q}_k = q_k + \frac{1}{|\mathcal{B}_i(q)|} \sum_{j \in \mathcal{B}_i(q)} \delta_j$ for all $k \in N \setminus \mathcal{B}_i(q)$, then

$$D_i(\bar{q}) = D_i(q).$$

The standard translation-invariance axiom is implied by the above condition; it corresponds to the case where the $\delta_j$ are equal for all $j \in \mathcal{B}_i(q)$.

Finally, we introduce two proportionality properties. As is the case for related axioms that appear in the literature on the measurement of income inequality, these requirements impose restrictions on the response of an index to specific replications of (parts of) the population. The first of these concerns the behavior of $D_i$ when the entire population is replicated for some specific functioning-failure profiles, and the second applies to replications of the set of those who are better off than $i$ for a fixed total population.

**Population proportionality.** For all $m \in \mathcal{N}$, for all $N, M \in \mathcal{P}_i$ such that $N \subset M$ and $|M| = m|N|$, for all $q \in \mathbb{R}^N_+$, for all $\bar{q} \in \mathbb{R}^M_+$ and for all $k \in N \setminus \{i\}$, if $\mathcal{B}_i(\bar{q}) = \mathcal{B}_i(q) = \{k\}$, $\bar{q}_k = q_k$, $q_j = q_i$ for all $j \in N \setminus \{k\}$ and $\bar{q}_j = q_i$ for all $j \in M \setminus \{k\}$, then

$$D_i(\bar{q}) = \frac{D_i(q)}{m^2}.$$

Population proportionality applies to specific situations where the population is replicated by a factor $m$ and the number of individuals with fewer functioning failures than $i$ is fixed at one. All other individuals have the same degree of functioning failure as $i$. Under these circumstances, the resulting replicated profile leads to a value of individual deprivation given by the value of the original profile divided by $m^2$. The reason why deprivation is divided by $m^2$ rather than $m$ is that there are two effects of an increased number of individuals. First, the relative importance of the deprivation caused by the single individual who has fewer functioning failures is diminished. Second, the number of individuals with whom $i$ can identify is multiplied.

**Deprivation proportionality.** For all $m \in \mathcal{N}$, for all $N \in \mathcal{P}_i$, for all $q, \bar{q} \in \mathbb{R}^N_+$ and for all $k \in N \setminus \{i\}$, if $\mathcal{B}_i(q) = \{k\} \subset \mathcal{B}_i(\bar{q})$, $|\mathcal{B}_i(\bar{q})| = m$, $\bar{q}_j = q_k$ for all $j \in \mathcal{B}_i(\bar{q})$, $q_j = q_i$ for all $j \in N \setminus \{k\}$ and $\bar{q}_j = q_j$ for all $j \in N \setminus \mathcal{B}_i(\bar{q})$, then

$$D_i(\bar{q}) = m^2 D_i(q).$$

Deprivation proportionality applies to situations where, for a fixed population, the number of those with fewer functioning failures than $i$ is replicated. Again, all other individuals have the same functioning failure as $i$. Because of the two effects of this replication, deprivation is required to be multiplied by $m^2$ as a consequence.

The members of the class of deprivation measures characterized by the above axioms are such that the degree of deprivation for a profile $q$ is obtained as the product of two terms with the following interpretation. The first factor is a multiple of the ratio of the number of agents who have fewer functioning failures than $i$ and the population size. This number is interpreted as an inverse indicator of agent $i$'s capacity to identify with other members of society. The second factor is the average of the differences between $q_i$ and the functioning failures of all agents in $\mathcal{B}_i(q)$. This part captures the aggregate alienation experienced by $i$ with respect to those who are better off.

To the best of our knowledge, this index of individual deprivation has not appeared in the literature before. Although our measure of individual deprivation, reinterpreted in terms of income distributions rather than distributions of functioning failures, resembles that suggested by Yitzhaki (1979), there is an important and substantial difference. Yitzhaki defines what we use as the second factor as the individual deprivation index; see Ebert and Moyes (2000) for a characterization. Thus, taking into consideration the lack of identification in addition to aggregate alienation is what distinguishes our approach from earlier contributions. The alienation-identification framework has been proposed in the income-distribution literature by Esteban and Ray (1994). Our deprivation measure, reinterpreted in terms of income distributions, resembles that of income polarization suggested by Esteban and Ray (1994). However, it distinguishes itself from the latter in that it is a measure of deprivation where an asymmetry in the alienation component is called for–an individual experiences alienation only with respect to those who are better off. Moreover, a more comprehensive concept of identification is required because an individual identifies not only with those like it but, instead, with all individuals who are equally well or worse off.

**Theorem 1.** *An individual measure of deprivation $D_i$ satisfies normalization, focus, conditional anonymity, homogeneity, strong translation invariance, population proportionality and deprivation proportionality if and only if there exists $\alpha_i \in \mathbb{R}_{++}$ such that, for all*

*$N \in \mathcal{P}_i$ and for all $q \in \mathbb{R}^N_+$,*

$$D_i(q) = 0 \quad \text{if } \mathcal{B}_i(q) = \emptyset$$

*and*

$$D_i(q) = \alpha_i \frac{|\mathcal{B}_i(q)|}{|N|^2} \sum_{j \in \mathcal{B}_i(q)} (q_i - q_j) \quad \text{if } \mathcal{B}_i(q) \neq \emptyset.$$

**Proof.** That the indices defined in the theorem statement possess the required properties is straightforward to verify.

Conversely, suppose $D_i$ satisfies the axioms of the theorem statement. First, consider a fixed population $N \in \mathcal{P}_i$, and let $q \in \mathbb{R}^N_+$. If $\mathcal{B}_i(q)$ is empty, normalization immediately implies $D_i(q) = 0$ as desired.

Now suppose that $\mathcal{B}_i(q) \neq \emptyset$. By definition of this set, this implies that $q_i$ is positive.

Consider first the case where $q$ is such that all agents in $\mathcal{B}_i(q)$ have a functioning failure of zero. By conditional anonymity, the identities of the individuals in $\mathcal{B}_i(q)$ are irrelevant and, therefore, we can consider the profile $(q_{-\mathcal{B}_i(q)}, 0\mathbf{1}_{|\mathcal{B}_i(q)|})$. By the focus axiom,

$$D_i(q_{-\mathcal{B}_i(q)}, 0\mathbf{1}_{|\mathcal{B}_i(q)|}) = D_i(q_i\mathbf{1}_{|N \setminus \mathcal{B}_i(q)|}, 0\mathbf{1}_{|\mathcal{B}_i(q)|}). \tag{1}$$

Using homogeneity with $\lambda = q_i$, it follows that

$$D_i(q_i\mathbf{1}_{|N \setminus \mathcal{B}_i(q)|}, 0\mathbf{1}_{|\mathcal{B}_i(q)|}) = q_i D_i(\mathbf{1}_{|N \setminus \mathcal{B}_i(q)|}, 0\mathbf{1}_{|\mathcal{B}_i(q)|}). \tag{2}$$

Now define

$$a_i(|\mathcal{B}_i(q)|, |N|) = D_i(\mathbf{1}_{|N \setminus \mathcal{B}_i(q)|}, 0\mathbf{1}_{|\mathcal{B}_i(q)|}).$$

Using (1) and substituting into (2), we obtain

$$D_i(q_{-\mathcal{B}_i(q)}, 0\mathbf{1}_{|\mathcal{B}_i(q)|}) = q_i a_i(|\mathcal{B}_i(q)|, |N|). \tag{3}$$

Because $\mathcal{B}_i(q)$ is non-empty, $a_i(|\mathcal{B}_i(q)|, |N|)$ is positive by normalization.

Now consider an arbitrary profile $q \in \mathbb{R}^N_+$. By the focus axiom, we can without loss of generality assume that $q_k = q_i$ for all $k \in N \setminus \mathcal{B}_i(q)$. Construct a profile $\bar{q} \in \mathbb{R}^N_+$ by letting $\bar{q}_j = q_j - q_j = 0$ for all $j \in \mathcal{B}_i(q)$ and $\bar{q}_k = q_i - \frac{1}{|\mathcal{B}_i(q)|} \sum_{j \in \mathcal{B}_i(q)} q_j$ for all $k \in N \setminus \mathcal{B}_i(q)$. Using strong translation invariance with $\delta_j = -q_j$ for all $j \in \mathcal{B}_i(q) = \mathcal{B}_i(\bar{q})$, it follows that $D_i(\bar{q}) = D_i(q)$. Because all agents in $\mathcal{B}_i(\bar{q})$ have a functioning failure of zero in $\bar{q}$, (3) implies

$$D_i(q) = D_i(\bar{q}) \quad = \quad \bar{q}_i a_i(|\mathcal{B}_i(\bar{q})|, |N|)$$

$$
\begin{aligned}
&= \left(q_i - \frac{1}{|\mathcal{B}_i(q)|} \sum_{j\in\mathcal{B}_i(q)} q_j\right) a_i(|\mathcal{B}_i(q)|, |N|) \\
&= \left(|\mathcal{B}_i(q)| q_i - \sum_{j\in\mathcal{B}_i(q)} q_j\right) \frac{a_i(|\mathcal{B}_i(q)|, |N|)}{|\mathcal{B}_i(q)|} \\
&= \frac{a_i(|\mathcal{B}_i(q)|, |N|)}{|\mathcal{B}_i(q)|} \sum_{j\in\mathcal{B}_i(q)} (q_i - q_j) \\
&= F_i(|\mathcal{B}_i(q)|, |N|) \sum_{j\in\mathcal{B}_i(q)} (q_i - q_j), \qquad (4)
\end{aligned}
$$

where $F_i(|\mathcal{B}_i(q)|, |N|) = a_i(|\mathcal{B}_i(q)|, |N|)/|\mathcal{B}_i(q)|$. $F_i$ is positive-valued because $a_i$ is.

Let $N \in \mathcal{P}_i$ be such that $|N| = 2$, let $m \in \mathcal{N}$ and let $q, \bar{q}$ be as in the definition of population proportionality. By (4), we have

$$D_i(q) = F_i(1,2)(q_i - q_k)$$

and

$$D_i(\bar{q}) = F_i(1,2m)(q_i - q_k).$$

By population proportionality,

$$F_i(1,2m)(q_i - q_k) = \frac{1}{m^2} F_i(1,2)(q_i - q_k)$$

and, because $q_k < q_i$, it follows that

$$F_i(1,2m) = \frac{1}{m^2} F_i(1,2)$$

for all $m \in \mathcal{N}$. Thus,

$$F_i(1,n) = \frac{4}{n^2} F_i(1,2) \qquad (5)$$

for all even $n \in \mathcal{N}$. Now suppose $n \in \mathcal{N}$ is odd. Let $m = 2$ and apply population proportionality again to obtain

$$F_i(1,2n) = \frac{1}{4} F_i(1,n).$$

Thus,

$$F_i(1,n) = 4F_i(1,2n).$$

Because $2n$ is even, (5) implies

$$F_i(1,2n) = \frac{1}{n^2} F_i(1,2)$$

and, therefore,

$$F_i(1,n) = 4F_i(1,2n) = \frac{4}{n^2}F_i(1,2)$$

for all odd $n \in \mathcal{N}$. Thus, letting $\alpha_i = 4F_i(1,2) \in \mathbb{R}_{++}$, we obtain

$$F_i(1,n) = \alpha_i \frac{1}{n^2} \tag{6}$$

for all $n \in \mathcal{N}$.

Let $N \in \mathcal{P}_i$ and $m \in \mathcal{N}$ be such that $m \leq n = |N|$ and consider $q, \bar{q}$ as in the definition of deprivation proportionality. Using (4), it follows that

$$D_i(q) = F_i(1,n)(q_i - q_k)$$

and

$$D_i(\bar{q}) = F_i(m,n)m(q_i - q_k).$$

By deprivation proportionality,

$$F_i(m,n)m(q_i - q_k) = m^2 F_i(1,n)(q_i - q_k)$$

and, therefore,

$$F_i(m,n) = mF_i(1,n).$$

Using (6), we obtain

$$F_i(m,n) = \alpha_i \frac{m}{n^2}.$$

Substituting into (4) completes the proof. ■

We now move on to aggregate measures of deprivation. An aggregate measure of deprivation is a function $\mathbf{D}\colon \Omega \to \mathbb{R}_+$. The interpretation is that $\mathbf{D}$ measures not only individual deprivation but the degree of deprivation in an entire society.

The first axiom we impose on $\mathbf{D}$ requires that if there are only two individuals, aggregate deprivation is given by the degree of individual deprivation suffered by the individual whose level of deprivation is higher, normalized by dividing by two—the number of agents. Of course, this requires that an individual index of deprivation $D_i$ is defined for all $i \in \mathbb{N}$. Because deprivation is a relative notion, at most one of the two individuals can have a positive level of deprivation and, consequently, this requirement is very reasonable: if there is only one individual who experiences a positive degree of deprivation, aggregate deprivation in a society should be determined by individual deprivation. If neither of the two individuals is deprived, they must have the same degree of functioning failure and, again, a natural property of an aggregate index is that aggregate deprivation is zero in this case. These considerations are expressed formally as follows.

**Two-person equivalence.** For all $j, k \in \mathbb{N}$ such that $j \neq k$ and for all $q \in \mathbb{R}_+^{\{j,k\}}$,

$$\mathbf{D}(q) = \frac{1}{2} \max\{D_j(q), D_k(q)\}. \tag{7}$$

The second axiom we employ in our characterization of an aggregate measure of deprivation is a rank-ordered recursivity property. It states that the value of the index for a population of at least size three can be obtained by first finding the aggregate deprivation of all agents but a worst-off (that is, one with maximal functioning failure) and then calculating the population-weighted average of this aggregate deprivation for the subgroup and the individual deprivation of the worst-off. Again, this axiom implicitly assumes that individual measures of deprivation are defined. This is a weak decomposability property because it is restricted to specific rank-ordered decompositions: only the individual deprivation of an agent who is worst-off in a profile can be treated separately from the others. This restriction is very intuitive because, by the focus axiom imposed on individual measures of deprivation, the other individual index values do not depend on anyone who is worst-off. We use the following notation in the formal statement of the axiom. For $N \in \mathcal{P}$ and $q \in \mathbb{R}_+^N$, we let $j_{\max} \in N$ be an agent in $N$ such that $q_{j_{\max}} \geq q_k$ for all $k \in N$, that is, $j_{\max}$ is an agent with maximal functioning failure (in case of ties, any agent can be chosen).

**Rank-ordered recursivity.** For all $N \in \mathcal{P}$ such that $|N| \geq 3$ and for all $q \in \mathbb{R}_+^N$,

$$\mathbf{D}(q) = \frac{(|N| - 1)}{|N|} \mathbf{D}(q_{-\{j_{\max}\}}) + \frac{1}{|N|} D_{j_{\max}}(q). \tag{8}$$

Finally, we impose a normalization axiom which incorporates an anonymity condition. It requires that a specific two-person functioning-failure profile leads to a level of aggregate deprivation of 1/2. Alternative normalizations could be employed, of course. Anonymity does not have to be required explicitly in our characterization of an aggregate measure of deprivation because, in the normalization axiom, the identities of the individuals involved are not specified. The main role of this axiom is to equalize the parameter values $\alpha_i$ of Theorem 1 in order to treat individuals impartially.

**Two-person normalization.** For all $j, k \in \mathbb{N}$ such that $j \neq k$ and for all $q \in \mathbb{R}_+^{\{j,k\}}$, if $q_j = 0$ and $q_k = 1$, then

$$\mathbf{D}(q) = 1/2.$$

Together with Theorem 1, the above axioms characterize a unique aggregate measure of deprivation.

**Theorem 2.** *An aggregate measure of deprivation* $\mathbf{D}$ *satisfies two-person equivalence, rank-ordered recursivity and two-person normalization with individual measures of deprivation* $D_i$ *that satisfy normalization, focus, conditional anonymity, homogeneity, strong translation invariance, population proportionality and deprivation proportionality if and only if, for all* $N \in \mathcal{P}$ *and for all* $q \in \mathbb{R}_+^N$,

$$\mathbf{D}(q) = 0 \quad \textit{if } \; \mathcal{B}_i(q) = \emptyset \textit{ for all } i \in N \tag{9}$$

*and*

$$\mathbf{D}(q) = \frac{1}{|N|^3} \sum_{i \in N : \mathcal{B}_i(q) \neq \emptyset} |\mathcal{B}_i(q)| \sum_{j \in \mathcal{B}_i(q)} (q_i - q_j) \quad \textit{if } \; \exists\, i \in N \textit{ such that } \mathcal{B}_i(q) \neq \emptyset. \tag{10}$$

**Proof.** That the aggregate measure defined in the theorem statement satisfies the axioms is easily verified. Now suppose $\mathbf{D}$ is an aggregate index of deprivation satisfying two-person equivalence, rank-ordered recursivity and two-person normalization and the $D_i$ are individual measures of deprivation satisfying the axioms of Theorem 1. By Theorem 1, we need to show that: (i) the aggregate measure of deprivation can be written as the arithmetic mean of the individual measures of deprivation; and (ii) the constants $\alpha_i$ of Theorem 1 must be equal to one.

To prove (i), we proceed by induction on the cardinality of $N$. Suppose $|N| = 2$. Let $N = \{j, k\}$ with $j \neq k$ and $q \in \mathbb{R}_+^N$. Without loss of generality, suppose $q_j \leq q_k$. It follows that $\mathcal{B}_j(q) = \emptyset$ and, by Theorem 1, $D_j(q) = 0$ and hence $D_k(q) \geq D_j(q)$. By two-person equivalence,

$$\mathbf{D}(q) = \frac{1}{2} \max\{D_k(q), D_j(q)\} = \frac{1}{2} D_k(q) = \frac{1}{2}(D_k(q) + D_j(q)) = \frac{1}{|N|} \sum_{i \in N} D_i(q).$$

Now suppose that the aggregate measure is equal to the arithmetic mean of the individual measures whenever $|N| \leq m$ for some $m \geq 2$. We have to establish that this is the case for $|N| = m + 1$ as well. Let $N$ be such that $|N| = m + 1$ and let $q \in \mathbb{R}_+^N$. By rank-ordered recursivity, (8) is true. By the induction hypothesis,

$$\mathbf{D}(q_{-\{j_{\max}\}}) = \frac{1}{(|N| - 1)} \sum_{i \in N \setminus \{j_{\max}\}} D_i(q_{-\{j_{\max}\}}). \tag{11}$$

By the focus axiom, $D_i(q_{-\{j_{\max}\}}) = D_i(q)$ for all $i \in N \setminus \{j_{\max}\}$. Using (11) and substituting into (8), we obtain the desired result.

To establish (ii), let $k \in \mathbb{N}$ be arbitrary. Choose any $j \in \mathbb{N} \setminus \{k\}$ and let $N = \{j, k\}$ and $q \in \mathbb{R}^N_+$ be such that $q_j = 0$ and $q_k = 1$. By Theorem 1 and part (i), $\mathbf{D}(q) = \alpha_k/2$. Two-person normalization requires that $\mathbf{D}(q) = 1/2$ which implies $\alpha_k = 1$. ■

Clearly, the minimal aggregate level of deprivation is equal to zero and attained in the case where everyone has the same level of functioning failure, that is, in the case of complete equality. This is true for Yitzhaki's (1979) deprivation index and for those of polarization as well if incomes are reinterpreted as indices of functionings. In contrast, the maximal level of Yitzhaki's deprivation index is attained for a distribution where one individual has access to all functionings and everyone else has the maximal possible functioning failure. Furthermore, Esteban and Ray's (1994) measure of polarization is maximal for a distribution where half of the population have full functioning failure whereas the other half have no functioning failures. Interestingly, our aggregate measure of deprivation is not maximal for either of those distributions (and an analogous observation applies to the aggregate measure of social exclusion characterized in the following section). In fact, it need not even be the case that maximal functioning failure and zero functioning failure are the only values attained in a distribution in order for this distribution to have maximal aggregate deprivation. To illustrate, we provide an example. Suppose that $N = \{1, \ldots, 13\}$ and the maximal possible functioning failure is equal to one. Consider a distribution $q \in \mathbb{R}^N_+$ such that $q_1 = \ldots = q_4 = 1$, $q_5 = q_6 = 1/2$, $q_7 = 1/4$ and $q_8 = \ldots = q_{13} = 0$. It is straightforward to verify that $\mathbf{D}(q) = 333.5/13^3$ and that this exceeds the value of $\mathbf{D}$ for any 13-dimensional distribution whose components assume the values zero and one only.

# 4 Individual and aggregate measures of social exclusion

To incorporate the dynamic aspect of social exclusion, we first consider an intertemporal extension of the individual measures of deprivation characterized in Theorem 1. We want to be able to calculate an individual measure of social exclusion for a data set with an arbitrary number of periods. For $i, t \in \mathbb{N}$ and $N \in \mathcal{P}_i$, let $\Gamma^t_{Ni} = (\mathbb{R}^N_+)^t$. Furthermore, for $i, t \in \mathbb{N}$, let $\Gamma^t_i = \cup_{N \in \mathcal{P}_i} \Gamma^t_{Ni}$. A $t$-period functioning-failure profile involving individual $i$ is a vector $\mathbf{q} = (q^1, \ldots, q^t) \in \Gamma^t_i$. For $\mathbf{q} \in \Gamma^1_i$, we obviously have $\mathbf{q} = (q^1)$ with

$q^1 \in \Omega_i$ and, thus, a one-period functioning-failure profile can be identified with a single functioning-failure profile as defined in the previous section. Note that, in a given profile, the population is the same in each time period. This is important because, given the temporal aspect of social exclusion, we need to take into account possible patterns of persistent deprivation over time for each individual. Defining $\Gamma_i = \cup_{t \in \mathbb{N}} \Gamma_i^t$, an individual measure of social exclusion for individual $i$ is a mapping $E_i \colon \Gamma_i \to \mathbb{R}_+$ that assigns $i$'s level of social exclusion to each profile of intertemporal functioning failures.

Our first axiom for an individual measure of social exclusion states that if there is only one time period, then the individual measure of social exclusion has the same value as the individual measure of deprivation–the two concepts coincide. This axiom is motivated by the view of social exclusion as deprivation over time adopted throughout this paper. As is the case for the axiom two-person equivalence employed in the previous section, this axiom requires the assumption that an individual index of deprivation $D_i$ is defined.

**Single-period equivalence.** For all $\mathbf{q} \in \Gamma_i^1$,

$$E_i(\mathbf{q}) = D_i(q^1).$$

The second axiom deals with the treatment of consecutive periods in which individual $i$ is in the group of those with minimal functioning failure. We require the index to be insensitive with respect to the number of consecutive periods in which the set of those with fewer functioning failures is empty. Analogously, if the set of individuals with fewer functioning failures than $i$ in the first or the last period is empty, the degree of social exclusion of individual $i$ is the same as that obtained without that period.

**Temporal independence.** For all $t \in \mathcal{N}$ and for all $\mathbf{q} \in \Gamma_i^t$,

(a) for all $\tau \in \{2, \ldots, t-1\}$, if $\mathcal{B}_i(q^{\tau-1}) = \mathcal{B}_i(q^\tau) = \emptyset$, then

$$E_i(\mathbf{q}) = E_i((q^1, \ldots, q^{\tau-1}, q^{\tau+1}, \ldots, q^t));$$

(b) if $\mathcal{B}_i(q^1) = \emptyset$, then

$$E_i(\mathbf{q}) = E_i((q^2, \ldots, q^t));$$

(c) if $\mathcal{B}_i(q^t) = \emptyset$, then

$$E_i(\mathbf{q}) = E_i((q^1, \ldots, q^{t-1})).$$

To illustrate the axiom, consider an example with $t = 4$. Suppose that $\mathcal{B}_i(q^2) = \mathcal{B}_i(q^3) = \emptyset$. Temporal independence requires that $E_i((q^1, q^2, q^3, q^4)) = E_i((q^1, q^2, q^4))$. The additional period in which no one has fewer functioning failures than $i$ does not matter, provided that this period immediately follows another period with that property. Given the assumption that persistence over time matters, one may wonder whether it might be more appropriate to require the index value to decrease if a period in which no one has fewer functioning failures than $i$ is added in the above-described fashion. However, the requirement that the index pays attention to persistence is already expressed by means of our last axiom (see below) and we do not want to over-emphasize the influence of this feature on the index value. Of course, alternative axioms could be justified as well, in which case the resulting class of measures may be different from that characterized here.

Our next axiom is a separability property. It requires that $E_i$ is additively decomposable in any two-subset partition of the periods if the two components of the partition are separated by a period in which $i$ has minimal functioning failure. We use the term 'separated' loosely here; as will become clear when we state the formal definition of the axiom, what is meant is that the last period of the first component of the partition is such that the set of those with fewer functioning failures than $i$ is empty. Additive decomposability is a standard axiom; see, for instance, Ebert and Moyes (2000) who use a suitably formulated version in a characterization of Yitzhaki's (1979) individual index of deprivation. We restrict the scope of the axiom to situations where the above-described separation property applies. This restriction is necessitated by our view that persistence over time should influence the degree of social exclusion and, thus, if there are consecutive periods where the set of those with fewer functioning failures is non-empty, the index should not have this additive structure.

**Conditional additive decomposability.** For all $t \in \mathcal{N}$, for all $\mathbf{q} \in \Gamma_i^t$ and for all $\tau \in \{1, \ldots, t-1\}$, if $\mathcal{B}_i(q^\tau) = \emptyset$, then

$$E_i(\mathbf{q}) = E_i((q^1, \ldots, q^\tau)) + E_i((q^{\tau+1}, \ldots, q^t)).$$

Finally, we formulate an axiom that ensures that the temporal pattern of the individual functioning failures is taken into consideration properly. As mentioned in the discussion of the previous axiom, additive decomposability is not a suitable condition if the phenomenon of persistence is to be captured. Instead, in situations where the set of those with fewer functioning failures is non-empty for all time periods under consideration, we require the average index value (where the average is taken over the number of periods) to be additively decomposable in the average values of social exclusion obtained for any two-subset partition of the periods. This requirement is one possibility of requiring a superadditive structure if there are consecutive periods in which the set of those with fewer functioning failures is non-empty. Again, alternative axioms could be employed but we think that ours is a plausible choice–one reason is its close relationship to the standard additive-decomposability condition.

**Conditional average decomposability.** For all $t \in \mathcal{N}$, for all $\mathbf{q} \in \Gamma_i^t$ and for all $\tau \in \{1, \ldots, t-1\}$, if $\mathcal{B}_i(q^s) \neq \emptyset$ for all $s \in \{1, \ldots, t\}$, then

$$\frac{1}{t}E_i(\mathbf{q}) = \frac{1}{\tau}E_i((q^1, \ldots, q^\tau)) + \frac{1}{t-\tau}E_i((q^{\tau+1}, \ldots, q^t)).$$

For all $t \in \mathbb{N}$ and for all $\mathbf{q} \in \Gamma_i^t$, let $\mathcal{T}_i(\mathbf{q})$ be the set of periods $\tau \in \{1, \ldots, t\}$ such that $\mathcal{B}_i(q^\tau) \neq \emptyset$. For any $t \in \mathbb{N}$ and for any profile $\mathbf{q} \in \Gamma_i^t$ such that $\mathcal{T}_i(\mathbf{q}) \neq \emptyset$, let $T_i^1(\mathbf{q})$ be the set of consecutive periods beginning with the first period $\tau \in \{1, \ldots, t\}$ such that $\mathcal{B}_i(q^\tau) \neq \emptyset$ and ending with the first period $\tau \in \{1, \ldots, t\}$ such that $\mathcal{B}_i(q^\tau) \neq \emptyset$ and $\mathcal{B}_i(q^{\tau+1}) = \emptyset$ if such a period exists; if not, the last period to be included in $T_i^1(\mathbf{q})$ is $t$. If $\mathcal{T}_i(\mathbf{q}) \cap [\{1, \ldots, t\} \setminus T_i^1(\mathbf{q})] \neq \emptyset$, the set $T_i^2(\mathbf{q})$ is obtained from $\{1, \ldots, t\} \setminus T_i^1(\mathbf{q})$ in the same way $T_i^1(\mathbf{q})$ is obtained from $\{1, \ldots, t\}$. Because $t$ is finite, this construction can be repeated until we obtain a partition $\{T_i^1(\mathbf{q}), \ldots, T_i^{\ell(\mathbf{q})}(\mathbf{q})\}$ of $\mathcal{T}_i(\mathbf{q})$, where $\ell(\mathbf{q}) \in \mathbb{N}$ is the number of sets of consecutive periods $\tau$ such that $\mathcal{B}_i(q^\tau)$ is non-empty.

Given an individual measure of deprivation $D_i$ satisfying the axioms of Theorem 1, the above axioms characterize the following individual index of social exclusion. Let $\mathbf{q} \in \Gamma_i$. If the set $\mathcal{T}_i(\mathbf{q})$ is empty, individual $i$ is in the set of agents with the lowest functioning failure in all periods and its level of social exclusion is equal to zero, the value of $i$'s deprivation in each period because $D_i$ satisfies normalization. If $\mathcal{T}_i(\mathbf{q})$ is non-empty, we consider all sets of consecutive periods $\tau$ in which $\mathcal{B}_i(q^\tau)$ is non-empty. Because we want to take into consideration the persistence in states of deprivation over time, we do not simply add up the levels of deprivation in each period but, instead, put a higher weight on situations where the state of deprivation persists over several periods. The weight is given, for each set of consecutive periods in which $i$ is deprived, by the number of these periods. These particular weights are implied by conditional average decomposability. For example, if there are ten periods and $\mathbf{q} \in \Gamma_i^{10}$ is such that $\mathcal{B}_i(q^3) = \mathcal{B}_i(q^4) = \mathcal{B}_i(q^8) = \mathcal{B}_i(q^{10}) = \emptyset$ and $\mathcal{B}_i(q^\tau) \neq \emptyset$ for all $\tau \in \{1, 2, 5, 6, 7, 9\} = \mathcal{T}_i(\mathbf{q})$, we have $\ell(\mathbf{q}) = 3$, $T_i^1(\mathbf{q}) = \{1, 2\}$, $T_i^2(\mathbf{q}) = \{5, 6, 7\}$, $T_i^3(\mathbf{q}) = \{9\}$ and the value of $E_i(\mathbf{q})$ is $2[D_i(q^1) + D_i(q^2)] + 3[D_i(q^5) + D_i(q^6) + D_i(q^7)] + D_i(q^9)$.

Combined with Theorem 1, the axioms imposed on $E_i$ characterize the following class of individual measures of social exclusion.

**Theorem 3.** *An individual measure of social exclusion $E_i$ satisfies single-period equivalence, temporal independence, conditional additive decomposability and conditional average* *decomposability with an individual measure of deprivation $D_i$ that satisfies normalization, focus, conditional anonymity, homogeneity, strong translation invariance, population proportionality and deprivation proportionality if and only if there exists $\alpha_i \in \mathbb{R}_{++}$ such that, for all $N \in \mathcal{P}_i$ and for all $\mathbf{q} \in \Gamma_{Ni}$,*

$$E_i(\mathbf{q}) = 0 \quad \textit{if} \;\; \mathcal{T}_i(\mathbf{q}) = \emptyset$$

*and*

$$E_i(\mathbf{q}) = \frac{\alpha_i}{|N|^2} \sum_{k=1}^{\ell(\mathbf{q})} |T_i^k(\mathbf{q})| \sum_{\tau \in T_i^k(\mathbf{q})} |\mathcal{B}_i(q^\tau)| \sum_{j \in \mathcal{B}_i(q^\tau)} (q_i^\tau - q_j^\tau) \quad \textit{if} \;\; \mathcal{T}_i(\mathbf{q}) \neq \emptyset.$$

**Proof.** That the indices defined in the theorem statement satisfy the axioms is straightforward to verify. Now suppose $D_i$ is an individual measure of deprivation satisfying the axioms of Theorem 1 and $E_i$ is an individual index of social exclusion satisfying the four axioms of the theorem statement.

First, consider profiles $\mathbf{q} \in \Gamma_i$ such that $T_i^1(\mathbf{q}) = \emptyset$. Thus, $\mathcal{B}_i(q^\tau) = \emptyset$ for all periods $\tau$. By repeated application of temporal independence, it follows that $E_i(\mathbf{q}) = E_i((q^1))$ and single-period equivalence implies $E_i(\mathbf{q}) = E_i((q^1)) = D_i(q^1)$. Because $\mathcal{B}_i(q^1) = \emptyset$ and $D_i$ satisfies normalization, it follows that $D_i(q^1) = 0$ and hence $E_i(\mathbf{q}) = 0$.

Now suppose $\mathbf{q} \in \Gamma_i$ is such that $T_i^1(\mathbf{q}) \neq \emptyset$. By Theorem 1, it is sufficient to prove that

$$E_i(\mathbf{q}) = \sum_{k=1}^{\ell(q)} |T_i^k(\mathbf{q})| \sum_{\tau \in T_i^k(\mathbf{q})} D_i(q^\tau);$$

the desired conclusion then follows immediately because the $D_i$ are assumed to satisfy the axioms of Theorem 1.

By temporal independence, we can without loss of generality assume that, in the first and last periods, the set of individuals with fewer functioning failures than $i$ is non-empty and, moreover, that there are $\ell(\mathbf{q}) \in \mathbb{N}$ sets $T_i^1(\mathbf{q}), \ldots, T_i^{\ell(\mathbf{q})}(\mathbf{q})$ of consecutive periods in which the set of those with fewer functioning failures than $i$ is non-empty where, if $\ell(\mathbf{q}) \geq 2$, any two consecutive sets $T_i^k$ and $T_i^{k+1}$ are separated by a single period $\tau_k$ in which $\mathcal{B}_i(q^{\tau_k})$ is empty.

Consider first the case where $\ell(\mathbf{q}) = 1$. We prove by induction on the cardinality of the set $T_i^1(\mathbf{q})$ that

$$E_i(\mathbf{q}) = |T_i^1(\mathbf{q})| \sum_{\tau \in T_i^1(\mathbf{q})} D_i(q^\tau). \tag{12}$$

If $|T_i^1(\mathbf{q})| = 1$, we have $T_i^1(\mathbf{q}) = \{1\}$ and $E_i(\mathbf{q}) = D_i(q^1) = |T_i^1(\mathbf{q})| \sum_{\tau \in T_i^1(\mathbf{q})} D_i(q^\tau)$ follows immediately from single-period equivalence. Now suppose (12) is true for all $|T_i^1(\mathbf{q})| \leq m$ for some $m \geq 1$. We have to show that (12) is true for $|T_i^1(\mathbf{q})| = m + 1$ as well. By conditional average decomposability,

$$\frac{1}{m+1} E_i(\mathbf{q}) = \frac{1}{m} E_i((q^1, \ldots, q^m)) + E_i((q^{m+1})).$$

By the induction hypothesis, $E_i((q^1, \ldots, q^m)) = m \sum_{\tau=1}^{m} D_i(q^\tau)$ and $E_i((q^{m+1})) = D_i(q^{m+1})$ and, therefore,

$$\frac{1}{m+1} E_i(\mathbf{q}) = \sum_{\tau=1}^{m+1} D_i(q^\tau)$$

which, because $|T_i^1(\mathbf{q})| = m + 1$, is equivalent to (12).

Finally, in the case where $\ell(\mathbf{q}) \geq 2$, the desired result follows immediately from repeated application of conditional additive decomposability and (12). ■

The final step in our axiomatic approach consists of obtaining an aggregate measure of social exclusion from the individual measures defined in Theorem 3. For $N \in \mathcal{P}$ and $t \in \mathbb{N}$, let $\Gamma_N^t = (\mathbb{R}_+^N)^t$. Furthermore, for $N \in \mathcal{P}$, define $\Gamma_N = \cup_{t \in \mathbb{N}} \Gamma_N^t$. Finally, let $\Gamma = \cup_{N \in \mathcal{P}} \Gamma_N$. We define an aggregate measure of social exclusion as a function $\mathbf{E}\colon \Gamma \to \mathbb{R}_+$ that assigns an aggregate level of social exclusion to each intertemporal profile of functioning failures $\mathbf{q} \in \Gamma$.

Our first axiom requires that an aggregate measure of social exclusion depend on the individual levels of social exclusion only and, moreover, that the identities of the individuals are irrelevant–only the individual levels of social exclusion experienced by the members of society matter.

**Anonymous individual invariance.** For all $N, M \in \mathcal{P}$ such that $|N| = |M|$, for all bijections $\rho\colon M \to N$, for all $\mathbf{q} \in \Gamma_N$ and for all $\bar{\mathbf{q}} \in \Gamma_M$, if $E_j(\bar{\mathbf{q}}) = E_{\rho(j)}(\mathbf{q})$ for all $j \in M$, then

$$\mathbf{E}(\bar{\mathbf{q}}) = \mathbf{E}(\mathbf{q}).$$

Note that an analogous condition is not required in our characterization of an aggregate measure of deprivation–the extent to which individual deprivation measures enter the desiderata for an aggregate measure is much more limited and restricted to specific two-person situations. The aggregation problem for social exclusion is considerably more complex because of a lack of separability caused by the observation that different individuals may be deprived in different periods. As a consequence, it is not possible to impose an axiom analogous to rank-ordered recursivity in the intertemporal setting.

The next axiom is an additivity condition. Its scope is restricted to situations in which the individual levels of social exclusion can be generated by a vector of profiles each of which is associated with a positive individual level of social exclusion for a single individual only.

**Conditional exclusion additivity.** For all $N \in \mathcal{P}$, for all $\mathbf{q} \in \Gamma_N$ and for all $(\mathbf{q}_i)_{i \in N}$ with $\mathbf{q}_i \in \Gamma_N$ for all $i \in N$, if $E_i(\mathbf{q}_i) = E_i(\mathbf{q})$ and $E_j(\mathbf{q}_i) = 0$ for all $i \in N$ and for all $j \in N \setminus \{i\}$, then

$$\mathbf{E}(\mathbf{q}) = \sum_{i \in N} \mathbf{E}(\mathbf{q}_i).$$

Next, we define a condition that reflects a unanimity principle formulated in terms of individual indices of social exclusion. If everyone in society experiences the same degree of social exclusion, then the aggregate level of social exclusion should be equal to that value as well.

**Exclusion identity.** For all $N \in \mathcal{P}$, for all $\mathbf{q} \in \Gamma_N$ and for all $e \in \mathbb{R}_+$, if $E_i(\mathbf{q}) = e$ for all $i \in N$, then

$$\mathbf{E}(\mathbf{q}) = e.$$

Finally, we impose a normalization condition analogous to that defined for an aggregate measure of deprivation in the previous section. Again, its major role is to equalize the constants $\alpha_i$.

**Two-person one-period normalization.** For all $j, k \in \mathbb{N}$ such that $j \neq k$ and for all $\mathbf{q} \in \Gamma^1_{\{j,k\}}$, if $q^1_j = 0$ and $q^1_k = 1$, then

$$\mathbf{E}(\mathbf{q}) = 1/2.$$

Together with our earlier results, we can now characterize the aggregate measure of social exclusion that is of particular interest in this paper.

**Theorem 4.** *An aggregate measure of social exclusion* $\mathbf{E}$ *satisfies anonymous individual invariance, conditional exclusion additivity, exclusion identity and two-person one-period normalization with individual measures of deprivation* $D_i$ *that satisfy normalization, focus, conditional anonymity, homogeneity, strong translation invariance, population proportionality and deprivation proportionality and with individual measures of social exclusion* $E_i$ *that satisfy single-period equivalence, temporal independence, conditional additive decomposability and conditional average decomposability if and only if, for all* $N \in \mathcal{P}$ *and for all* $\mathbf{q} \in \Gamma_N$*,*

$$\mathbf{E}(\mathbf{q}) = 0 \quad \text{if } \mathcal{T}_i(\mathbf{q}) = \emptyset \text{ for all } i \in N \tag{13}$$

*and*

$$\mathbf{E}(\mathbf{q}) = \frac{1}{|N|^3} \sum_{i \in N: \mathcal{T}_i(q) \neq \emptyset} \sum_{k=1}^{\ell(\mathbf{q})} |T_i^k(\mathbf{q})| \sum_{\tau \in T_i^k(\mathbf{q})} |\mathcal{B}_i(q^\tau)| \sum_{j \in \mathcal{B}_i(q^\tau)} (q_i^\tau - q_j^\tau) \tag{14}$$

*if there exists* $i \in N$ *such that* $\mathcal{T}_i(\mathbf{q}) \neq \emptyset$.

**Proof.** That the aggregate measure defined in the theorem statement satisfies the axioms is straightforward to verify. Now suppose the individual measures of deprivation $D_i$ satisfy normalization, focus, conditional anonymity, homogeneity, strong translation invariance, population proportionality and deprivation proportionality, the individual measures of social exclusion $E_i$ satisfy single-period equivalence, temporal independence, conditional additive decomposability and conditional average decomposability, and $\mathbf{E}$ is an aggregate index of deprivation satisfying anonymous individual invariance, conditional exclusion additivity, exclusion identity and two-person one-period normalization. In light of Theorems 1 and 3, it remains to show that: (i) the aggregate measure of social exclusion can be written as the arithmetic mean of the individual measures of social exclusion; and (ii) the constants $\alpha_i$ of Theorem 1 are equal to one.

Let $N \in \mathcal{P}$. Setting $N = M$ and letting $\rho$ be the identity mapping in the definition of anonymous individual invariance, it follows that there exists a function $f\colon \mathbb{R}_+^N \to \mathbb{R}_+$ such that

$$\mathbf{E}(\mathbf{q}) = f\left((E_i(\mathbf{q}_i))_{i \in N}\right)$$

for all $\mathbf{q} \in \Gamma_N$. By anonymous individual invariance, this function is symmetric and does not depend on the identities of the individuals in $N$. By Theorem 3, for any $\mathbf{q} \in \Gamma_N$, there exists $(\mathbf{q}_i)_{i \in N}$ with $\mathbf{q}_i \in \Gamma_N$ for all $i \in N$ such that $E_i(\mathbf{q}_i) = E_i(\mathbf{q})$ and $E_j(\mathbf{q}_i) = 0$ for all $i \in N$ and for all $j \in N \setminus \{i\}$. Conditional exclusion additivity implies

$$\mathbf{E}(\mathbf{q}) = \sum_{i \in N} \mathbf{E}(\mathbf{q}_i)$$

for all $\mathbf{q} \in \Gamma_N$. For $i \in N$ and $\mathbf{q}_i \in \Gamma_N$, define $\mathbf{e}^i(\mathbf{q}_i) \in \mathbb{R}_+^N$ as follows. For all $j \in N$,

$$\mathbf{e}_j^i(\mathbf{q}_i) = \begin{cases} E_i(\mathbf{q}_i) & \text{if } j = i, \\ 0 & \text{if } j \in N \setminus \{i\}. \end{cases}$$

Substituting, we obtain

$$f\left((E_i(\mathbf{q}_i))_{i \in N}\right) = \sum_{i \in N} f\left(\mathbf{e}^i(\mathbf{q}_i)\right)$$

for all $\mathbf{q} \in \Gamma_N$. By exclusion identity and the symmetry of $f$, it follows that

$$e = f(e\mathbf{1}_{|N|}) = nf(e, 0\mathbf{1}_{|N|-1})$$

and, therefore, $f(e, 0\mathbf{1}_{|N|-1}) = e/|N|$ for all $e \in \mathbb{R}_+$. Substituting back, we obtain

$$f\left((E_i(\mathbf{q}_i))_{i\in N}\right) = \frac{1}{|N|}\sum_{i\in N} E_i(\mathbf{q})$$

and, using the definition of $f$, it follows that

$$\mathbf{E}(\mathbf{q}) = \frac{1}{|N|}\sum_{i\in N} E_i(\mathbf{q})$$

for all $\mathbf{q} \in \Gamma_N$, which completes the proof of (i). The proof of (ii) is analogous to that of part (ii) in Theorem 2, using two-person one-period normalization instead of two-period normalization. ■

The proof of (i) in the above theorem follows that of Aczél's (1966) Theorem 5.3.4 characterizing the arithmetic mean; see Aczél (1966, pp. 239–240).

# 5 An application to EU countries

The purpose of this section is to illustrate the aggregate measure of social exclusion, **E** as defined in (13) and (14), and the aggregate deprivation measure, **D** as defined in (9) and (10), using the European Community Household Panel (ECHP). We base our analysis on all the waves available of ECHP, which cover the period from 1994 to 2001. The surveys are conducted at a European national level. The ECHP is an ambitious effort at collecting information on the living standards of the households of the EU member states using common definitions, information collection methods and editing procedures. It contains detailed information on incomes, socio-economic characteristics, housing amenities, consumer durables, social relations, employment conditions, health status, subjective evaluation of well-being, etc. Of the 15 EU member states, we could not consider Austria, Finland, Luxembourg and Sweden since the data for these countries were not available for all the waves. For similar reasons we had to exclude Germany and the UK. In particular, the ECHP surveys of these countries were substituted by national surveys, SOEP and BHPS respectively, that did not collect information on all the variables considered in our application. The unit of our analysis is the individual. The calculation uses required sample weights and, since we are interested in analyzing the persistence of deprivation, we considered only individuals that were interviewed in all the waves. In ECHP a person's quality of life is measured along the following dimensions: financial difficulties, basic needs and consumption, housing conditions, durables, health, social contacts and participation, and life satisfaction.

For the choice of the non-monetary indicators to be considered for measuring social exclusion and deprivation with the ECHP, we follow some suggestions of Eurostat (2000) and analyze the well-being of EU societies focusing on the following non-monetary variables:

- Financial difficulties: 1. Persons living in households that have great difficulties in making ends meet; 2. Persons living in households that are in arrears with (re)payment of housing or utility bills;
- Basic necessities: 3. Persons living in households which cannot afford meat, fish or chicken every second day; 4. Persons living in households which cannot afford to buy new clothes; 5. Persons living in households which cannot afford a week's holiday away from home;
- Housing conditions: 6. Persons living in the accommodation without a bath or shower; 7. Persons living in the dwelling with damp walls, floors, foundations, etc.; 8. Persons living in households which have a shortage of space;
- Durables: 9. Persons not having access to a car due to a lack of financial resources in the household; 10. Persons not having access to a telephone due to a lack of financial resources in the household; 11. Persons not having access to a color TV due to a lack of financial resources in the household.

The individual functioning failure employed in the application is the number, unweighted, of the above listed 11 variables that the interviewed claimed to have, or not to have, depending on the variable. As a clarifying example for the way we obtain functioning failures, consider the variables in the first category. An individual living in a household that has great difficulties in making ends meet is assigned a score of 1; if, in addition, it lives in a household that is in arrears with (re)payment of housing or utility bills, then it obtains a score of 2; if, furthermore, it is unable to afford meat, fish or chicken every second day, then it receives the score 3. As mentioned in Section 2, the approach proposed in this paper is much more flexible because it allows for alternative aggregation procedures.

In order to examine the robustness of our index and the differences between the measure of deprivation proposed here and other comparable measures in the literature, we use the same data to compute the Yitzhaki (1979) index of deprivation (Table 1 and Figure 1) and the polarization index of Esteban and Ray (1994), with the parameter indicating the sensitivity to polarization equal to 1 (Table 3 and Figure 3). Note that the indices are computed using individual functioning failures and not incomes. Hence the ranking of the countries could differ from the usual results.

Numerical estimates of social exclusion as measured by **E** for the EU member states are reported in Table 4, the values being plotted in Figure 4.

The first column of all the tables gives the names of the countries for whom the required information was available. In the following columns we present, for each country, the estimates for the various indices and, in parentheses, we indicate the ranking of the country.

Several interesting features emerge from Table 1 and Table 2 (values plotted in Figure 1 and 2 respectively). Portugal is the most deprived country followed by Greece. At a distance we observe the other two Southern European countries, namely Spain and Italy. The value of deprivation for Ireland is slightly higher than the one for Italy in the early periods under analysis. The countries appear to be grouped into three classes according to the level of deprivation reached: Portugal and Greece into the first; Ireland, Spain and Italy into the second; all the remaining countries into the remaining class. The grouping is more pronounced with the Yitzhaki measure of deprivation, the distinction between the last two groups being less sharp with our index. Throughout, we observe a declining trend and convergence over time, in particular of the second and third groups. Thus, values of deprivation are much more similar in the last wave than they were in the first.

The effect of the introduction of the lack of identification in our index is to lower the level of deprivation as compared to the Yitzhaki index. However, the two indices do not display major differences: the rankings of the countries are mostly the same, with a few exceptions (Spain and Ireland in 1995 and 1996, and Spain and Italy in 1999). These findings allow us to conclude that ours is a robust measure of deprivation in that it is largely consistent with an alternative index.

Polarization and deprivation are two very different phenomena, even when the same identification-alienation framework is applied. As emerges from the comparison of both Tables 1 and 2 with Table 3 (values plotted in Figures 1 and 2, and in Figure 3). All the countries exhibit more similar values in polarization as opposed to deprivation; Ireland, and not Portugal, is on average the most polarized country, followed by Italy. Denmark is confirmed to be the country with the lowest value, with few exceptions in the analyzed years.

Estimates of social exclusion are reported in Table 4, the values being plotted in Figure 4. Portugal is the most excluding country followed by Greece. As in the case of deprivation, at a distance we observe the other two Southern European countries, namely, Spain and Italy. The value of **E** for Ireland is slightly higher than the one for Italy. If we consider the ranking of countries from high to low social exclusion, then an unambiguous sequence is Portugal, Greece, Spain, Ireland, Italy, France, Belgium, the Netherlands and Denmark. In the exclusion measure, as opposed to deprivation, we do consider persistence in the deprivation state. Persistence is the key variable in understanding the different performance of EU member states in the two measures suggested in this paper. Portugal and Greece present a greater dissimilarity in social exclusion than in deprivation in all the years considered. This fact is caused by the higher persistence in the deprivation state that individuals face in Portugal than in Greece. In other words, in each period the percentage of the population that is deprived is slightly higher in Portugal than in Greece, but in the latter it is easier for individuals to escape from the deprivation state than it is in the former. Hence, the individuals deprived that we observe in each period vary more over time in Greece than in Portugal.

# 6 Conclusion

This paper investigates deprivation and social exclusion both from a theoretical perspective and from an empirical viewpoint. We present an axiomatic approach that identifies attractive measures of social exclusion and clarifies their possible relationship to the measurement of deprivation. In addition, we apply our measures to recent EU data. We conclude the paper with a few suggestions for future work.

On the theoretical side, it would be interesting to investigate the interplay between measures of social exclusion and other social indicators such as mobility and polarization; see, for instance, Wolfson (1994), Esteban and Ray (1994, 1999), Akerlof (1997), Fields and Ok (1999), Wang and Tsui (2000) and Duclos, Esteban and Ray (2004). Furthermore, systematic approaches to ethically significant social-exclusion measurement analogous to those employed in the measurement of inequality (see, for example, Kolm, 1969; Atkinson, 1970; Sen, 1973; Ebert, 1987) could be developed.

From an empirical perspective, the analysis carried out with our data could be complement by studies employing observations from other countries. Moreover, an analysis of the possible determinants of the different measures, such as education, age and other individual and societal characteristics, would be an interesting task.

Phenomena of deprivation and social exclusion are essentially multifaceted. However, in the analytical framework we have assumed, as a primitive, an index of the individual multidimensional functioning failures ($q_i$). While this choice seems justified in the present paper (where the focus is on the axiomatic characterization of new measures of deprivation and social exclusion), this leaves another task as a subject of future research. It would be interesting to axiomatize suitable methods of aggregating the individual functioning failures into an index of these failures, or to treat the functioning failures themselves as the primitives for the measurement of deprivation and social exclusion without necessarily deriving functioning-failure aggregates as an intermediate step.

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Table 1: Yitzhaki Deprivation in EU Member States (1994-2001).

| | 1994 | 1995 | 1996 | 1997 | 1998 | 1999 | 2000 | 2001 |
|---|---|---|---|---|---|---|---|---|
| **Belgium** | (7) 0.563 | (7) 0.552 | (7) 0.497 | (7) 0.459 | (7) 0.456 | (7) 0.458 | (7) 0.434 | (7) 0.448 |
| **Denmark** | (8) 0.532 | (9) 0.417 | (9) 0.408 | (9) 0.347 | (9) 0.360 | (9) 0.357 | (9) 0.343 | (9) 0.305 |
| **Greece** | (2) 1.331 | (2) 1.211 | (2) 1.188 | (2) 1.148 | (2) 1.037 | (2) 1.045 | (2) 1.009 | (2) 0.991 |
| **Spain** | (4) 0.877 | (3) 0.828 | (3) 0.832 | (3) 0.811 | (3) 0.733 | (3) 0.671 | (4) 0.614 | (4) 0.627 |
| **France** | (6) 0.667 | (6) 0.615 | (6) 0.607 | (6) 0.583 | (6) 0.562 | (6) 0.544 | (5) 0.516 | (5) 0.494 |
| **Ireland** | (3) 0.944 | (4) 0.805 | (4) 0.804 | (4) 0.723 | (5) 0.618 | (5) 0.614 | (6) 0.465 | (6) 0.474 |
| **Italy** | (5) 0.767 | (5) 0.778 | (5) 0.728 | (5) 0.669 | (4) 0.680 | (4) 0.664 | (3) 0.637 | (3) 0.653 |
| **Netherlands** | (9) 0.466 | (8) 0.430 | (8) 0.442 | (8) 0.424 | (8) 0.382 | (8) 0.373 | (8) 0.380 | (8) 0.381 |
| **Portugal** | (1) 1.350 | (1) 1.320 | (1) 1.250 | (1) 1.242 | (1) 1.192 | (1) 1.150 | (1) 1.069 | (1) 1.060 |

Table 2: Deprivation in EU Member States (1994-2001).

| | 1994 | 1995 | 1996 | 1997 | 1998 | 1999 | 2000 | 2001 |
|---|---|---|---|---|---|---|---|---|
| **Belgium** | (7) 0.457 | (7) 0.450 | (7) 0.417 | (7) 0.385 | (7) 0.385 | (7) 0.386 | (7) 0.366 | (7) 0.382 |
| **Denmark** | (8) 0.435 | (9) 0.337 | (9) 0.337 | (9) 0.290 | (9) 0.306 | (9) 0.297 | (9) 0.292 | (9) 0.257 |
| **Greece** | (2) 0.991 | (2) 0.916 | (2) 0.887 | (2) 0.866 | (2) 0.797 | (2) 0.798 | (2) 0.767 | (2) 0.756 |
| **Spain** | (4) 0.669 | (4) 0.635 | (4) 0.634 | (3) 0.625 | (3) 0.564 | (4) 0.514 | (4) 0.479 | (4) 0.491 |
| **France** | (6) 0.527 | (6) 0.487 | (6) 0.487 | (6) 0.469 | (6) 0.453 | (6) 0.443 | (5) 0.419 | (5) 0.403 |
| **Ireland** | (3) 0.742 | (3) 0.644 | (3) 0.639 | (4) 0.592 | (5) 0.508 | (5) 0.511 | (6) 0.386 | (6) 0.399 |
| **Italy** | (5) 0.609 | (5) 0.624 | (5) 0.584 | (5) 0.531 | (4) 0.532 | (3) 0.526 | (3) 0.505 | (3) 0.527 |
| **Netherlands** | (9) 0.395 | (8) 0.367 | (8) 0.383 | (8) 0.368 | (8) 0.335 | (8) 0.321 | (8) 0.325 | (8) 0.330 |
| **Portugal** | (1) 1.018 | (1) 1.002 | (1) 0.948 | (1) 0.948 | (1) 0.903 | (1) 0.871 | (1) 0.800 | (1) 0.801 |

**Table 3: Esteban and Ray Polarization in EU Member States (1994-2001).**

| | 1994 | 1995 | 1996 | 1997 | 1998 | 1999 | 2000 | 2001 |
|---|---|---|---|---|---|---|---|---|
| **Belgium** | (4) 0.330 | (4) 0.328 | (5) 0.322 | (7) 0.309 | (7) 0.309 | (7) 0.310 | (7) 0.300 | (7) 0.308 |
| **Denmark** | (5) 0.326 | (9) 0.288 | (9) 0.287 | (9) 0.263 | (9) 0.270 | (9) 0.267 | (9) 0.262 | (9) 0.242 |
| **Greece** | (8) 0.317 | (3) 0.340 | (4) 0.332 | (3) 0.331 | (2) 0.338 | (2) 0.338 | (1) 0.340 | (2) 0.339 |
| **Spain** | (7) 0.317 | (7) 0.321 | (6) 0.322 | (6) 0.325 | (5) 0.326 | (6) 0.320 | (3) 0.321 | (3) 0.329 |
| **France** | (3) 0.332 | (5) 0.328 | (3) 0.333 | (4) 0.330 | (4) 0.326 | (5) 0.326 | (4) 0.319 | (6) 0.313 |
| **Ireland** | (1) 0.364 | (1) 0.360 | (1) 0.363 | (1) 0.370 | (1) 0.353 | (1) 0.354 | (6) 0.308 | (5) 0.320 |
| **Italy** | (2) 0.356 | (2) 0.356 | (2) 0.353 | (2) 0.336 | (3) 0.330 | (3) 0.334 | (2) 0.331 | (1) 0.342 |
| **Netherlands** | (9) 0.314 | (8) 0.301 | (8) 0.311 | (8) 0.303 | (8) 0.283 | (8) 0.276 | (8) 0.279 | (8) 0.281 |
| **Portugal** | (6) 0.321 | (6) 0.326 | (7) 0.322 | (5) 0.329 | (6) 0.324 | (4) 0.327 | (5) 0.318 | (4) 0.324 |

**Table 4: Social Exclusion in EU Member States (1994-2001).**

| | |
|---|---|
| **Belgium** | (7) 17.379 |
| **Denmark** | (9) 11.855 |
| **Greece** | (2) 46.765 |
| **Spain** | (3) 31.149 |
| **France** | (6) 24.148 |
| **Ireland** | (4) 30.896 |
| **Italy** | (5) 28.384 |
| **Netherlands** | (8) 16.720 |
| **Portugal** | (1) 59.670 |

Figure 1: Yitzhaki Deprivation in EU Member States (1994-2001).

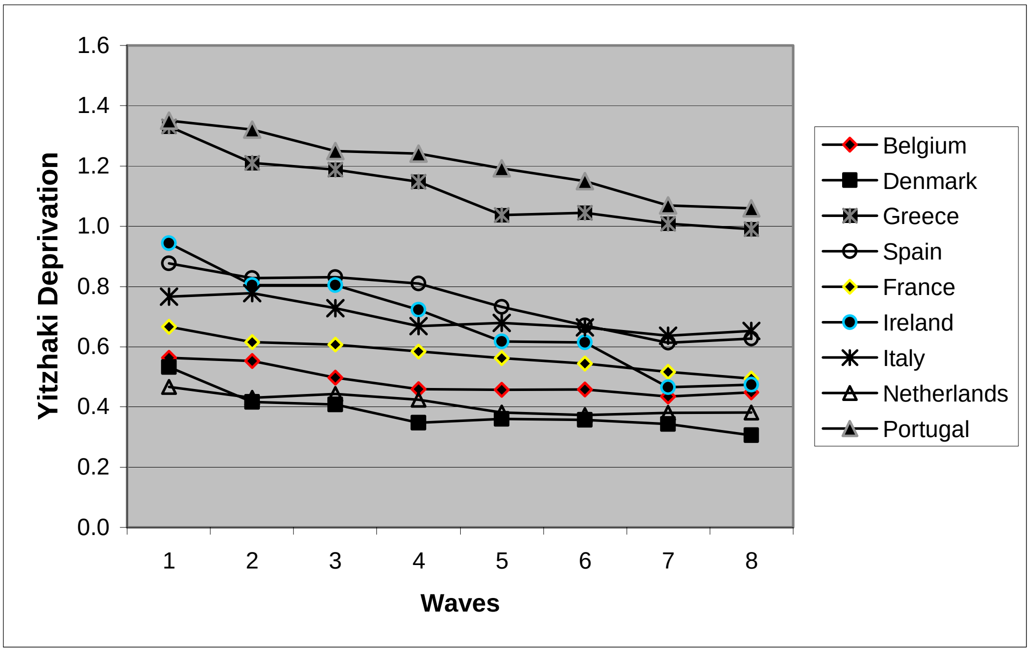


Figure 2: Deprivation in EU Member States (1994-2001).

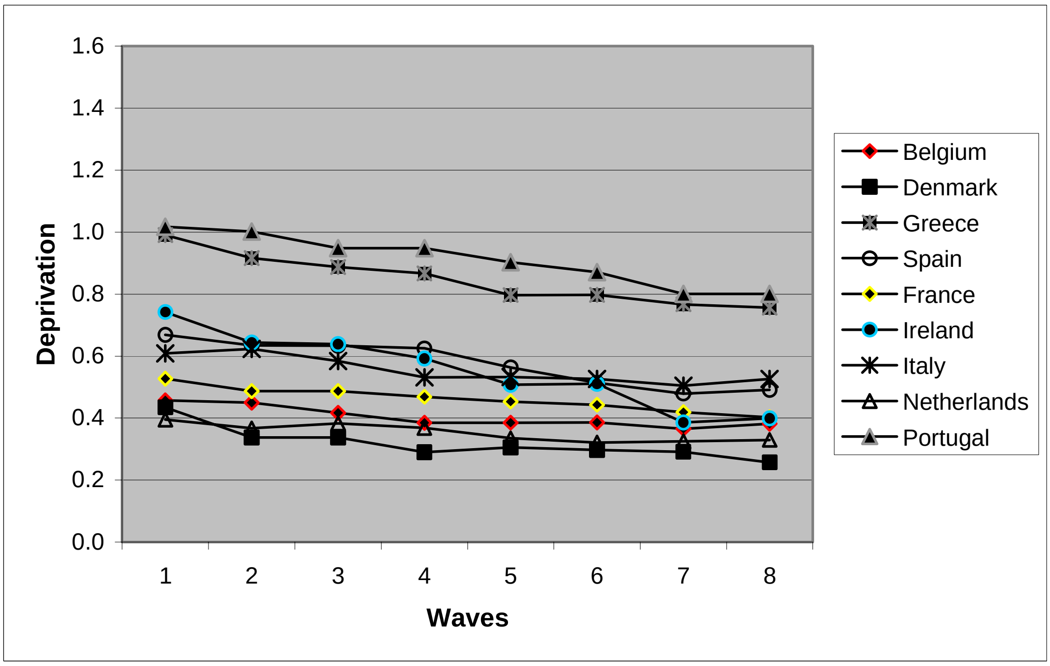

**Figure 3: Esteban and Ray Polarization in EU Member States (1994-2001).**

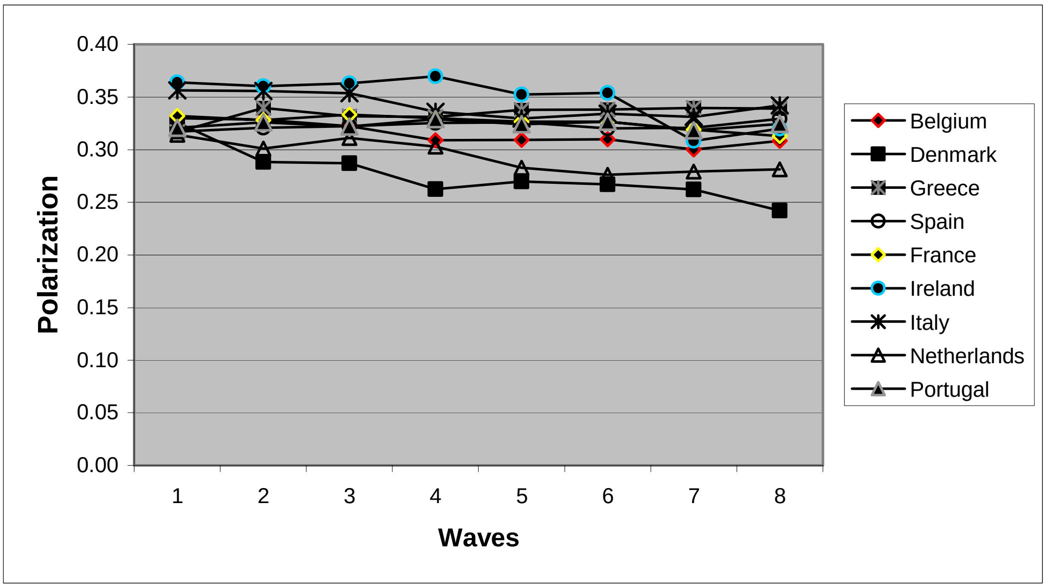


**Figure 4: Social Exclusion in EU Member States (1994-2001).**